Direct Multitarget Tracking and Multisensor Fusion Using Antenna Arrays

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Abstract: This paper investigates a direct Target Motion Analysis (TMA) estimator for the problem of calculating the states (i.e. source positions, velocities, etc.) of multiple sources from measurements made with multiple (fixed or moving) antenna arrays. We use the novel Subspace Data Fusion (SDF) approach and extend it to the multisensor case. In the SDF approach, subspaces are formed in a first pre-processing step from the raw antenna outputs. Then, the parameters of interest are estimated directly from a single cost function, which results from fusing all subspaces. This approach requires only a single low-dimensional optimization and completely circumvents the bearing data association problem inherent in traditional TMA approaches. We derive the Cramér-Rao Bound (CRB) for the direct multitarget tracking problem. In Monte Carlo simulations we find that the SDF estimator approaches the CRB and *always* performs better than or equal to the traditional TMA approach. We show that the state estimation accuracy can be improved by using multiple antenna arrays.

1 Introduction

The estimation of the state of multiple emitting sources using passive sensors is a widely investigated problem encountered in various fields like wireless communication, radar, and sonar. This problem is commonly referred as the Target Motion Analysis (TMA) problem. Bearing measurements collected by multiple fixed direction finding (DF) sensors or taken from points along the trajectory of a single moving observer can be used to determine the target states. If the target is stationary, the bearings can be intersected to determine the emitter location (sometimes called triangulation). Various aspects of the two-dimensional and three-dimensional Bearings-only Tracking (BOT) problem examined in the literature include estimation algorithms, estimation accuracy, and target observability [Bec01].

Here, we consider a three-dimensional scenario with Q inertially (i.e. non-accelerating) moving targets and P observers each moving along an arbitrary but known trajectory and equipped with an antenna array (commonly used to solve the DF problem). The sensors must not be time-synchronized to calculate Times of Arrival (TOAs), because we focus on the BOT problem. At N different points in space, the p-th sensor receives signals of all sources and collects batches of antenna outputs, $p = 1, ..., P$. The scenario is assumed to be stationary during one batch and non-stationary from batch to batch.

Within the traditional approach to the multiple source TMA problem, first of all for each batch of each sensor Directions of Arrival (DOAs) of all sources at all points in space are estimated with a DF method like the subspace-based Multiple Signal Classification (MUSIC) method [Sch86]. The subsequent measurement-to-track (M2T) association step consists of partitioning the DOAs into sets of DOAs, or tracks, belonging to the same source. Then, the DOA tracks of all sensors are fused in a track-to-track (T2T) association step. Finally, the DOAs for each source are used to determine its state with the help of a suitable BOT algorithm [Bec01, NLG84]. We will consider the Least Squares solution of the Maximum Likelihood (ML) estimator which requires the variances of the DOAs as well, but is asymptotically efficient. Fig. 1 shows the basic steps of a conventional bearings-only TMA system.

It is well-known that all tracking algorithms lead to track loss whenever the DOAs of the targets cannot be resolved for a longer period of time. Multiple Hypotheses Tracking (MHT) is generally accepted as the preferred method for solving the M2T association problem in modern multitarget tracking systems [Bla04]. MHT can deal with cases where the Global Nearest Neighbor (GNN) approach or the Joint Probabilistic Data Association (JPDA) fail. However, in situations where the variances of the measurements are too large, even MHT is unable to partition the sensor data correctly. Another drawback originates from the bias which is always present in the DOA estimates for a finite amount of data, number of array elements or signal-to-noise ratio (SNR) [FBL04, XB92]. Moreover, the bearing estimates may be more or less correlated, a fact that is not considered in the traditional approach.

Recently, some direct position determination (DPD) methods based on the antenna outputs have been proposed without computing intermediate parameters like DOAs. The basic idea for a subspace-based DPD approach goes back to the pioneering work of Wax and Kailath [WK85a]. They noted that in this way the data association step is avoided. Moreover, this kind of approach was used for a multiarray network in order to estimate the

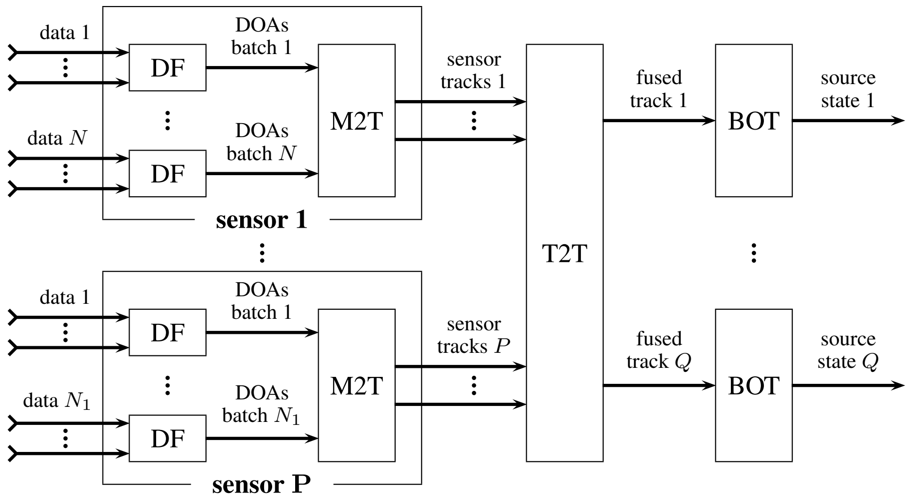

Figure 1: Basic steps of the traditional TMA approach

positions of multiple sources without explicitly computing DOAs and TOAs [WA06]. ML methods can be found e.g. in [Wei04, AW07], but they are more computationally demanding in the case of multiple sources. The DPD approach can be adapted to estimate DOAs and DOA rates [WE95].

In our previous work, we proposed a subspace-based DPD approach for a single moving array [DOR08]. Moreover, we have shown that the DPD approach can be extended to estimate the target states (e.g. positions, velocities) [OD08] and adapted to solve the bearing data association problem in the presence of clutter by using a fictitious array [ODW08]. Furthermore, we found that it is preferable to use a high-dimensional search in the case of multiple sources with intermittent emission [Ois09b] and we proposed a direct DPD approach to determine the total number of emitting sources [Ois09a]. In all these Subspace Data Fusion (SDF) approaches, the parameters of interest are obtained by minimizing a single cost function into which all subspaces at all sensor positions enter jointly (Fig. 2).

In this paper, we show that the SDF approach offers the advantage for the multitarget multisensor case that the M2T and T2T association problem inherent in the traditional method is circumvented. Furthermore, this approach is computationally efficient, as all source states are assessed from the minima of one common MUSIC-type cost function that depends on as many parameters as there are degrees of freedom for a single source. Moreover, the accuracy of the state estimates is much better compared with the traditional TMA approach in situations where the variance of DOA estimates deviates from the Cramér-Rao Bound (CRB), e.g. in the case of a weak source, closely-spaced sources or crossing DOA trajectories. We show that the state estimation accuracy can be improved by using additional (fixed) sensors.

The paper is organized as follows: In Section 2 we consider the multisensor TMA problem. In Section 2.1 we present the data model, in Section 2.2 we formulate the problem, and in Section 2.3, we derive the CRB for the TMA problem based on the received signals. Then,

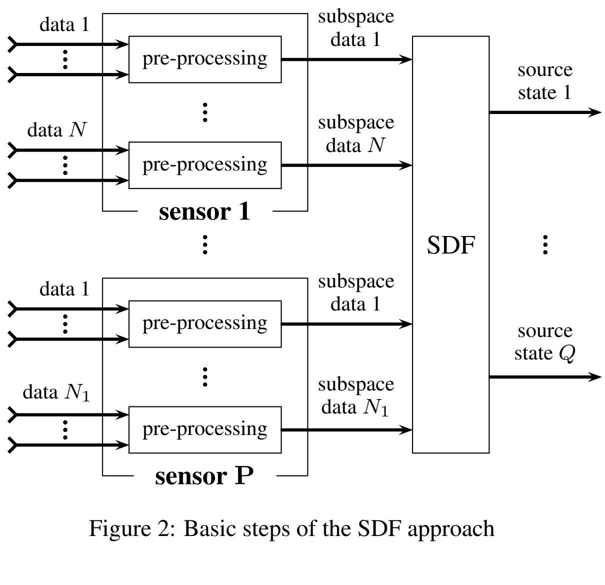

Figure 2: Basic steps of the SDF approach

in Section 3.1, we outline the traditional TMA approach, and in Section 3.2, we give a brief review of the novel SDF approach. In Section 4 we present Monte Carlo simulation results that demonstrate the estimator's performance. The conclusions are given in Section 5.

The following notations are used throughout this paper: $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively; $\mathbf{I}_n$ and $\mathbf{0}_n$ denote the $n \times n$-dimensional identity and zero matrix, respectively; and $\mathrm{E}\{\cdot\}$ denotes the expectation operation.

2 Estimation Problem

We consider P (fixed or moving) antenna arrays and Q inertially moving sources in the far field of the arrays. The sources are assumed to radiate narrowband signals (i.e. the source bandwidth is much smaller than the reciprocal of the time delay across the array) with wavelengths centered around a common wavelength λ. Let the q-th source state, $q = 1, ..., Q$, be comprised in the Cartesian position-velocity vector $\mathbf{x}_q = (\mathbf{p}_q^T(t_0), \dot{\mathbf{p}}_q^T)^T \in \mathbb{R}^{6\times 1}$, where $\mathbf{p}_q(t_0) = (x_{0,q}, y_{0,q}, z_{0,q})^T$ denotes the source position at reference time t_0 and $\dot{\mathbf{p}}_q = (\dot{x}_q, \dot{y}_q, \dot{z}_q)^T$ denotes the constant velocity. The source position at some time t is related to the source state $\mathbf{x}_q$ by

$$\mathbf{p}_q(t) = \mathbf{p}_q(t_0) + (t - t_0)\,\dot{\mathbf{p}}_q\,. \tag{1}$$

Fig. 3 shows the geometry for the scenario of Q inertially moving sources and a single sensor moving along an arbitrary but known trajectory. During the movement of the p-th array, N batches of data are collected at time $t_{p,n}$ at the positions $\mathbf{r}_{p,n} := \mathbf{r}_p(t_{p,n})$, $n = 1, ..., N$. For the sake of simplicity, we assume that the antenna attitude does not change with time, i.e. the orientation of the sensor-fixed coordinate system is fixed during the batches. The geometry between the p-th observer, $p = 1, ..., P$, and the q-th source, $q = 1, ..., Q$, at time $t_{p,n}$, $n = 1, ..., N$, is given by the Cartesian relative vector

$$\begin{aligned}\triangle\mathbf{r}_{p,n}(\mathbf{x}_{0,q}) &= \mathbf{r}_{p,n} - \mathbf{p}_q(t_{p,n}) \\ &= \begin{pmatrix} \triangle x_{p,n,q} \\ \triangle y_{p,n,q} \\ \triangle z_{p,n,q} \end{pmatrix} = \triangle r_{p,n,q} \begin{pmatrix} \sin\alpha_{p,n,q}\cos\varepsilon_{p,n,q} \\ \cos\alpha_{p,n,q}\cos\varepsilon_{p,n,q} \\ \sin\varepsilon_{p,n,q} \end{pmatrix}\,,\end{aligned} \tag{2}$$

where $(\triangle r_{p,n,q}, \alpha_{p,n,q}, \varepsilon_{p,n,q})$ denote the corresponding spherical coordinates (i.e. distance, azimuth angle and elevation angle). They are given by the nonlinear relations

$$\begin{aligned}\triangle r_{p,n,q} &= \sqrt{\triangle x_{p,n,q}^2 + \triangle y_{p,n,q}^2 + \triangle z_{p,n,q}^2}\,, \\ \alpha_{p,n,q} &= \arctan\frac{\triangle x_{p,n,q}}{\triangle y_{p,n,q}}\,, \\ \varepsilon_{p,n,q} &= \arctan\frac{\triangle z_{p,n,q}}{\sqrt{\triangle x_{p,n,q}^2 + \triangle y_{p,n,q}^2}}\,.\end{aligned} \tag{3}$$

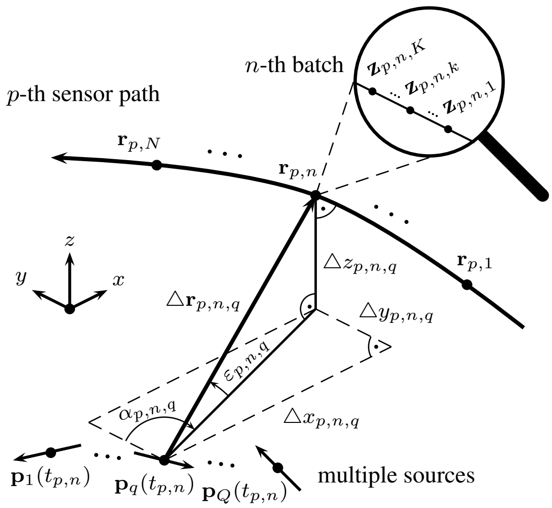

Figure 3: Geometry for the scenario of multiple inertially moving sources and the p-th sensor

2.1 Data Model

We assume that the antenna arrays are identical and each composed of M elements. In the presence of additional noise, the received vector $\mathbf{z}_p(t) \in \mathbb{C}^{M\times 1}$ observed by the p-th array can be expressed as

$$\mathbf{z}_p(t) = \sum_{q=1}^{Q} \mathbf{a}_p(\mathbf{p}_q(t))\, s_{p,q}(t) + \mathbf{w}_p(t)\,, \tag{4}$$

where $s_{p,q}(t)$ is the signal transmitted by the q-th source at time t, where $\mathbf{w}_p(t)$ represents the noise, and where the array transfer vector $\mathbf{a}_p(\mathbf{p})$ expresses its complex response to a planar wavefront arriving from the position $\mathbf{p}$. For each observation at time $t_{p,n}$, let the array be sampled sequentially at $k = 1, ..., K$ different mutually exclusive time slots. Then, the sampled version of the signals in Eq. 4 is given by

$$\begin{aligned} \mathbf{z}_{p,n,k} &:= \mathbf{z}_p(t_{p,n} + (k-1)\,T)\,, \\ s_{p,n,k,q} &:= s_{p,q}(t_{p,n} + (k-1)\,T)\,, \\ \mathbf{w}_{p,n,k} &:= \mathbf{w}_p(t_{p,n} + (k-1)\,T)\,. \end{aligned} \tag{5}$$

The time T between two snapshots is assumed to be much smaller (several orders in magnitude) than the time interval between two time slots. Therefore, the array transfer vectors can be considered quasistatic in each slot, i.e. the sensor's displacement during each time slot is negligible. Consequently,

$$\mathbf{a}_{p,n}(\mathbf{x}_{0,q}) := \mathbf{a}_p(\,\mathbf{p}_q(t_{p,n} + (k-1)\,T)\,) \tag{6}$$

does not depend on k.

External and blind array calibration techniques are well-known, e.g. the calibration of an airborne antenna array is described in [MSHK07]. We assume that the antenna array is perfectly calibrated for which the array transfer vector is a known function of the source states:

$$\mathbf{a}_{p,n}(\mathbf{x}_{0,q}) = \begin{pmatrix} \mathrm{e}^{j\,\mathbf{k}_{p,n}^T(\mathbf{x}_{0,q})\,\mathbf{d}_1} \\ \vdots \\ \mathrm{e}^{j\,\mathbf{k}_{p,n}^T(\mathbf{x}_{0,q})\,\mathbf{d}_M} \end{pmatrix} . \tag{7}$$

The array transfer vector depends on the position $\mathbf{d}_m$ of the m-th antenna element, $m = 1, ..., M$, relative to the position $\mathbf{r}_{p,n}$ and the wavenumber vector

$$\mathbf{k}_{p,n}(\mathbf{x}_{0,q}) = \frac{2\pi}{\lambda}\,\frac{\triangle \mathbf{r}_{p,n}(\mathbf{x}_{0,q})}{\triangle r_{p,n,q}} . \tag{8}$$

Alternatively, the array transfer vector may be parameterized by the DOA: $\mathbf{a}(\alpha_{p,n,q}, \varepsilon_{p,n,q})$. By substituting Eq. 2 into Eq. 8, the corresponding wavenumber vector follows

$$\mathbf{k}(\alpha_{p,n,q}, \varepsilon_{p,n,q}) = \frac{2\pi}{\lambda} \begin{pmatrix} \sin\alpha_{p,n,q}\cos\varepsilon_{p,n,q} \\ \cos\alpha_{p,n,q}\cos\varepsilon_{p,n,q} \\ \sin\varepsilon_{p,n,q} \end{pmatrix} . \tag{9}$$

The array data model (Eq. 4) can be written more compactly as

$$\mathbf{z}_{p,n,k} = \mathbf{A}_{p,n}(\boldsymbol{\rho}_{\mathbf{x}})\,\mathbf{s}_{p,n,k} + \mathbf{w}_{p,n,k} , \tag{10}$$

where $\mathbf{A}_{p,n}(\boldsymbol{\rho}_{\mathbf{x}}) = [\mathbf{a}_{p,n}(\mathbf{x}_{0,1}) \cdots \mathbf{a}_{p,n}(\mathbf{x}_{0,Q})] \in \mathbb{C}^{M\times Q}$ is the array transfer matrix, all source states are comprised in the vector $\boldsymbol{\rho}_{\mathbf{x}} = (\mathbf{x}_{0,1}^T, ..., \mathbf{x}_{0,Q}^T)^T \in \mathbb{R}^{6Q\times 1}$, and $\mathbf{s}_{p,n,k} = (s_{p,n,k,1}, ..., s_{p,n,k,Q})^T \in \mathbb{C}^{Q\times 1}$ is a signal vector formed from the emitted signals.

We introduce the compact data model

$$\mathbf{z}_{p,k} = \boldsymbol{\mathcal{A}}_p(\boldsymbol{\rho}_{\mathbf{x}})\,\mathbf{s}_{p,k} + \mathbf{w}_{p,k} \tag{11}$$

by stacking the vectors on top and using a block-diagonal matrix:

$$\begin{aligned} \mathbf{z}_{p,k} &= (\mathbf{z}_{p,1,k}^T, ..., \mathbf{z}_{p,N,k}^T)^T \in \mathbb{C}^{MN\times 1} , \\ \boldsymbol{\mathcal{A}}_p(\boldsymbol{\rho}_{\mathbf{x}}) &= \mathrm{diag}[\mathbf{A}_{p,1}(\boldsymbol{\rho}_{\mathbf{x}}) \cdots \mathbf{A}_{p,N}(\boldsymbol{\rho}_{\mathbf{x}})] \in \mathbb{C}^{MN\times QN} , \\ \mathbf{s}_{p,k} &= (\mathbf{s}_{p,1,k}^T, ..., \mathbf{s}_{p,N,k}^T)^T \in \mathbb{C}^{QN\times 1} , \\ \mathbf{w}_{p,k} &= (\mathbf{w}_{p,1,k}^T, ..., \mathbf{w}_{p,N,k}^T)^T \in \mathbb{C}^{MN\times 1} . \end{aligned}$$

2.2 Problem Statement

The received data batches depend on the array transfer vectors, which depend on the relative vectors, which themselves depend on the desired source states. Now, the problem is stated as follows: Estimate all source states $\boldsymbol{\rho}_{\mathbf{x}}$ from all received signals $\mathbf{z}_{p,k}$, $p = 1, ..., P$, $k = 1, ..., K$. To solve the multiple source TMA problem, we make the following assumptions:

A1. The noise vectors $\mathbf{w}_{p,k}$, $p = 1, ..., P$, $k = 1, ..., K$, (Eq. 4) are zero-mean complex Gaussian and temporally and spatially uncorrelated with the covariance

$$\begin{aligned} \mathrm{E}\left\{\mathbf{w}_{p,k}\,\mathbf{w}_{p',k'}^{H}\right\} &= \sigma_{\mathrm{w}}^{2}\,\mathbf{I}_{MN}\,\delta_{p,p'}\,\delta_{k,k'}\,, \\ \mathrm{E}\left\{\mathbf{w}_{p,k}\,\mathbf{w}_{p',k'}^{T}\right\} &= \mathbf{0}_{MN}\,, \end{aligned} \tag{12}$$

where $\delta_{a,b}$ denotes the Kronecker delta.

A2. The signal vectors $\mathbf{s}_{p,n,k}$, $p = 1, ..., P$, $n = 1, ..., N$, $k = 1, ..., K$, (Eq. 4) are fixed and need to be estimated (deterministic data model). This does not exclude the possibility that the signals are sampled from a random process. Moreover, we assume that $\sum_{k=1}^{K} \mathbf{s}_{p,n,k}\,\mathbf{s}_{p,n,k}^{H}$ is positive definite.

A3. The number of source signals Q is constant and known. In the past, several methods have been proposed to determine the number of signal sources [WK85b].

2.3 Cramér-Rao Bound

For judging an estimation problem, it is important to know the maximum estimation accuracy that can be attained with all given measurements $\mathbf{Z}$. Moreover, since the CRB is a lower bound for any unbiased estimator, its parameter dependencies reveal characteristic features of the estimation problem. Then, the CRB is related to the covariance matrix $\mathbf{C}$ of the estimation error $\triangle\boldsymbol{\rho}_{\mathbf{x}} = \boldsymbol{\rho}_{\mathbf{x}} - \hat{\boldsymbol{\rho}}_{\mathbf{x}}(\mathbf{Z})$ of any unbiased estimator $\hat{\boldsymbol{\rho}}_{\mathbf{x}}(\mathbf{Z})$ as

$$\mathbf{C} = \mathrm{E}\left\{\triangle\boldsymbol{\rho}_{\mathbf{x}}\,\triangle\boldsymbol{\rho}_{\mathbf{x}}^{T}\right\} \geq \mathrm{CRB}(\boldsymbol{\rho}_{\mathbf{x}})\,, \tag{13}$$

where the inequality means that the matrix difference is positive semidefinite. If the estimator attains the CRB then it is called efficient.

The target parameters of the p-th sensor are comprised in the vector

$$\boldsymbol{\rho}_{p} = \left(\bar{\mathbf{s}}_{p,1}^{T}, \tilde{\mathbf{s}}_{p,1}^{T}, ..., \bar{\mathbf{s}}_{p,K}^{T}, \tilde{\mathbf{s}}_{p,K}^{T}, \boldsymbol{\rho}_{\mathbf{x}}^{T}\right)^{T} \in \mathbb{R}^{Q(2NK+6)\times 1}\,, \tag{14}$$

where overbar and overtilde indicate the real and imaginary part of the source signals. The CRB is given by the inverse Fisher Information Matrix (FIM), i.e. $\mathrm{CRB}(\boldsymbol{\rho}) = \mathrm{FIM}^{-1}(\boldsymbol{\rho})$ with

$$\mathrm{FIM}(\boldsymbol{\rho}_{p}) = \mathrm{E}\left\{\left(\frac{\partial\mathcal{L}(\mathbf{Z}_{p};\boldsymbol{\rho}_{p})}{\partial\boldsymbol{\rho}_{p}}\right)\left(\frac{\partial\mathcal{L}(\mathbf{Z}_{p};\boldsymbol{\rho}_{p})}{\partial\boldsymbol{\rho}_{p}}\right)^{T}\right\}\,, \tag{15}$$

where $\mathbf{Z}_p = [\mathbf{z}_{p,1} \cdots \mathbf{z}_{p,K}]$ are all measurements of the p-th sensor and

$$\mathcal{L}(\mathbf{Z}_p; \boldsymbol{\rho}_p) = -KMN \ln(\pi\sigma_{\mathrm{w}}^2) - \frac{1}{\sigma_{\mathrm{w}}^2} \sum_{k=1}^{K} \left|\mathbf{z}_{p,k} - \boldsymbol{\mathcal{A}}_p(\boldsymbol{\rho}_{\mathbf{x}})\,\mathbf{s}_{p,k}\right|^2 , \tag{16}$$

is the log-likelihood function. In this log-likelihood function $\mathbf{z}_{p,k}$, $k = 1, ..., K$, are random variables due to the random variables $\mathbf{w}_{p,k}$, $k = 1, ..., K$, and the expectation operation in Eq. 15 is w.r.t. these random variables.

Performing all calculations analog to [OD08, SN89, YB92], we obtain the FIM for the p-th sensor on the source states after some algebra (Assumption A1):

$$\mathrm{FIM}_p(\boldsymbol{\rho}_{\mathbf{x}}) = \frac{2}{\sigma_{\mathrm{w}}^2} \sum_{k=1}^{K} \mathrm{Re}\left\{\boldsymbol{\mathcal{S}}_{p,k}^H\,\mathbf{D}_p^H\,\mathbf{P}_{\boldsymbol{\mathcal{A}}_p}^{\perp}\,\mathbf{D}_p\,\boldsymbol{\mathcal{S}}_{p,k}\right\} \tag{17}$$

with

$$\begin{aligned}
\boldsymbol{\mathcal{S}}_{p,k} &= \mathbf{I}_{6Q} \otimes \mathbf{s}_{p,k} \in \mathbb{C}^{6NQ^2 \times 6Q} , \\
\mathbf{D}_p &= [\mathbf{D}_{p,1} \cdots \mathbf{D}_{p,Q}] \in \mathbb{C}^{MN \times 6NQ^2} , \\
\mathbf{D}_{p,q} &= \left[\frac{\partial \boldsymbol{\mathcal{A}}_p}{\partial x_{0,q}}, \frac{\partial \boldsymbol{\mathcal{A}}_p}{\partial y_{0,q}}, \frac{\partial \boldsymbol{\mathcal{A}}_p}{\partial z_{0,q}}, \frac{\partial \boldsymbol{\mathcal{A}}_p}{\partial \dot{x}_q}, \frac{\partial \boldsymbol{\mathcal{A}}_p}{\partial \dot{y}_q}, \frac{\partial \boldsymbol{\mathcal{A}}_p}{\partial \dot{z}_q}\right] \in \mathbb{C}^{MN \times 6NQ} , \\
\mathbf{P}_{\boldsymbol{\mathcal{A}}_p}^{\perp} &= \mathbf{I}_{MN} - \boldsymbol{\mathcal{A}}_p\,(\boldsymbol{\mathcal{A}}_p^H \boldsymbol{\mathcal{A}}_p)^{-1} \boldsymbol{\mathcal{A}}_p^H \in \mathbb{C}^{MN \times MN} ,
\end{aligned}$$

where $\otimes$ denotes the Kronecker product.

It is assumed that the measurements are independent from sensor to sensor. The resulting CRB is

$$\mathrm{CRB}(\boldsymbol{\rho}_{\mathbf{x}}) = \Big[\mathrm{FIM}_1(\boldsymbol{\rho}_{\mathbf{x}}) + ... + \mathrm{FIM}_P(\boldsymbol{\rho}_{\mathbf{x}})\Big]^{-1} . \tag{18}$$

The CRB expression (Eq. 17 and Eq. 18) is quite complicated, and it is difficult to see how the bound on the estimation accuracy is affected by the different parameters. For this reason we remark, that the CRB for the multiple source TMA problem based on the received data batches depends on the number of sensors P, the sensor parameters (i.e. noise variance σ_{w}^2, number of array elements M, array geometry $\mathbf{d}_m$, $m = 1, ..., M$, and number of collected samples K), the number of sources Q, the number of batches per sensor N, the emitted signals $\mathbf{s}_{p,k}$, $k = 1, ..., K$, and the geometry between sensor and source $\triangle\mathbf{r}_{p,n,q}$, $p = 1, ..., P$, $n = 1, ..., N$, $q = 1, ..., Q$.

3 TMA Approaches

In this section, we present the implementation of the investigated estimators: the traditional TMA approach (Fig. 1) and the SDF approach (Fig. 2). For the DF step of the traditional approach, we use the well-known subspace-based MUSIC algorithm [Sch86], because in this way the same pre-processing step is applied to the sensor data, so that both approaches can be compared equally.

In the **pre-processing step**, subspaces are calculated for each batch of each sensor by performing an eigendecomposition of the covariance matrix (Assumptions A2 and A3):

$$\mathbf{R}_{p,n} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{z}_{p,n,k}\, \mathbf{z}_{p,n,k}^{H} = \bar{\mathbf{U}}_{p,n}\, \bar{\mathbf{\Lambda}}_{p,n}\, \bar{\mathbf{U}}_{p,n}^{H} + \mathbf{U}_{p,n}\, \mathbf{\Lambda}_{p,n}\, \mathbf{U}_{p,n}^{H}\,, \tag{19}$$

where the column vectors of $\bar{\mathbf{U}}_{p,n} \in \mathbb{C}^{M \times Q}$ and $\mathbf{U}_{p,n} \in \mathbb{C}^{M \times M-Q}$ are the eigenvectors spanning the signal and noise subspaces of the covariance $\mathbf{R}_{p,n}$, respectively, with the associated eigenvalues in decreasing order on the diagonals of $\bar{\mathbf{\Lambda}}_{p,n} \in \mathbb{R}^{Q \times Q}$ and $\mathbf{\Lambda}_{p,n} \in \mathbb{R}^{M-Q \times M-Q}$, respectively.

3.1 Traditional TMA Approach

The traditional approach is divided into four steps (Fig. 1): the DF step, the M2T step, the T2T step, and the BOT step.

In the **DF step**, the subspace-based MUSIC method is used to determine the DOAs by minimizing the projection of the array transfer vector onto the noise subspace [Sch86]. For the n-th batch of the p-th sensor, $n = 1, ..., N$, $p = 1, ..., P$, the inverse MUSIC cost function reads

$$f_{\mathrm{MUSIC},p,n}^{-1}(\alpha, \varepsilon) = \mathbf{a}^{H}(\alpha, \varepsilon)\, \mathbf{U}_{p,n}\, \mathbf{U}_{p,n}^{H}\, \mathbf{a}(\alpha, \varepsilon)\,, \tag{20}$$

where the array transfer vector (Eq. 7) is parameterized by the DOA (α, ε) (Eq. 9). The DOA estimates are given by the locations of the Q smallest values of the cost function. It is well-known that MUSIC gives a superior resolution and tends to be power independent compared to other DF methods (e.g. Capon's method, conventional beamforming).

In the **M2T** and **T2T step**, the bearing data association problem is solved by partitioning the DOAs into sets of DOAs originating from the same source and fusing the resulting DOA tracks from all sensors. The bearing data association is not figured out, because in Section 4 we consider an ideal data association, but we note that this can be achieved e.g. by MHT. However, finally we obtain the DOA track of the q-th source, $q = 1, ..., Q$: $\hat{\alpha}_{p,n,q}, \hat{\varepsilon}_{p,n,q}$, $p = 1, ..., P$, $n = 1, ..., N$.

For the **BOT step**, we assume that the DOA errors are zero-mean Gaussian and that the DOAs are mutually uncorrelated, uncorrelated from sensor to sensor, from target to target, and from observation to observation. Then, the cost function for the q-th source has a Least Squares form and reads

$$f_{\mathrm{BOT},q}(\mathbf{x}) = \sum_{p=1}^{P} \left[\sum_{n=1}^{N} \frac{[\hat{\alpha}_{p,n,q} - \alpha_{p,n,q}(\mathbf{x})]^2}{\sigma_{\alpha,p,n,q}^2} + \sum_{n=1}^{N} \frac{[\hat{\varepsilon}_{p,n,q} - \varepsilon_{p,n,q}(\mathbf{x})]^2}{\sigma_{\varepsilon,p,n,q}^2} \right]\,, \tag{21}$$

where $\sigma^2_{\alpha,p,n,q}$, $\sigma^2_{\varepsilon,p,n,q}$ are the variances of the DOAs, and where the bearing errors correspond to the physical angle differences in $[0, \pi]$. The expected DOAs are parameterized by the source state $\mathbf{x}$ (Eq. 3). Now, the q-th source state is obtained by finding the global minimum of Eq. 21. We note that the DOA variances are unknown and need to be estimated, because otherwise an estimator could be used with even further reduced performance, and that the DOAs may be more or less correlated, but these correlations are not considered in the BOT approach.

3.2 SDF Approach

In this section, we outline the SDF approach to solve the TMA problem (Fig. 2). This approach relies on the same key idea as the localization approach of Wax and Kailath for decentralized array processing [WK85a]. They mentioned that this kind of estimation offers the advantage that the association problem inherent to the traditional method is circumvented. Furthermore, no intermediate parameters like DOAs or additional parameters like DOA variances are necessary.

The SDF approach is based on the same sequence of noise subspaces. We can therefore use the same pre-processing step applied to the array output data. The SDF approach uses a MUSIC-type cost function [Sch86], which minimizes the sum of all projections of the array transfer vectors at the sensor positions onto the corresponding noise subspaces. The source states are calculated directly in one step by fusing the subspaces of all batches of all sensors:

$$f_{\mathrm{SDF}}(\mathbf{x}) = \sum_{p=1}^{P} \sum_{n=1}^{N} \mathbf{a}^H_{p,n}(\mathbf{x})\, \mathbf{U}_{p,n}\, \mathbf{U}^H_{p,n}\, \mathbf{a}_{p,n}(\mathbf{x})\,, \tag{22}$$

where the array transfer vector (Eq. 7) is parameterized by the source state $\mathbf{x}$ (Eq. 8). The cost function shows minima for a proper choice of $\mathbf{x}$, if the subspace of each sensor and each batch is orthogonal to the corresponding array transfer vector.

4 Simulation Results

As an illustration, Monte Carlo simulations with 1500 runs have been carried out to study the performance of the estimators given in Sections 3.1 and 3.2. In our simulations, we use a suitable optimization to find the minima of the cost functions (Eq. 20, Eq. 21, and Eq. 22) and initialize every search with the true value.

We consider a scenario with $P = 2$ sensors and $Q = 2$ targets shown in Fig. 4 (left). The first sensor moves along an arc from $(-1, 0, 1)$ km to $(0, -1, 1)$ km and the second sensor is fixed at $(-0.5, 1, 1)$ km. We assume that each sensor collects (only) $N = 16$ batches at time t_n with $K = 100$ samples per batch, and that $t_0 = t_N$. For each sensor, we consider a 10-element uniform circular antenna array with element positions $\mathbf{d}_m =$

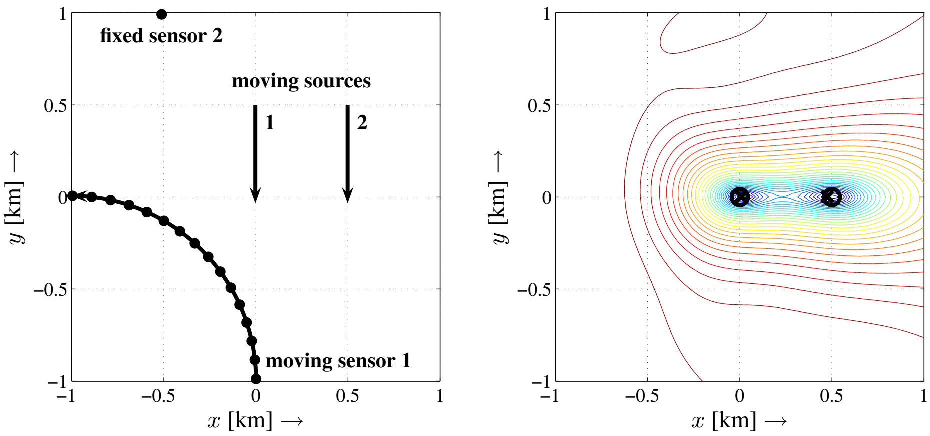

Figure 4: Left: Considered scenario; Right: xy-plane of the SDF cost function with true (circles) and estimated (crosses) target locations

$\rho\,(\cos\frac{m\pi}{5}, \sin\frac{m\pi}{5}, 0)^T$ and radius $\rho = \frac{\lambda}{2}(\sin\frac{\pi}{10})^{-1}$. The two ground-located sources move inertially from $(0, 0.5, 0)$ km to $(0, 0, 0)$ km, and parallel from $(0.5, 0.5, 0)$ km to $(0.5, 0, 0)$ km, respectively. For all emitted source signals, we use in our simulations the assumption of constant amplitude for each batch of each sensor: $|s_{p,n,k,q}| = s$. Then, the single element signal-to-noise ratio of a single source is defined by $\mathrm{SNR} = s^2/\sigma_{\mathrm{w}}^2$.

For the sake of simplicity, we give the traditional approach a head start, but we show that nevertheless the SDF approach performs always better or equal: Firstly, we assume an ideal M2T and T2T association. Secondly, we exploit the CRB on the azimuth and elevation angles [SN89, YB92] for the DOA variances in Eq. 21: $\sigma_{\alpha,n,q,p} = 1/\sin\varepsilon_{n,q,p}$ and $\sigma_{\varepsilon,n,q,p} = 1/\cos\varepsilon_{n,q,p}$.

Note, that if we use only the data of the second sensor, the problem has not a unique solution, because the observability condition established in [Bec93] is not satisfied. With the assumption that the sensor lies always above each source ($\triangle z_{p,n,q} > 0$, $p = 1, ..., P$, $n = 1, ..., N$, $q = 1, ..., Q$), the considered TMA problem has a unique solution, if we use the first sensor or both sensors, because the observability condition and the condition for unique DF [WZ89] hold. For the multisensor case and $\mathrm{SNR} = 5$ dB, the xy-plane of the SDF cost function (Eq. 22) is shown in Fig. 4 (right). The cost function displays well-pronounced minima and no further spurious peaks.

In Fig. 5, we show only the root mean square error (RMSE) of the x-location of the first source ($x_{0,1}$-coordinate), because the RMSE of the other coordinates, even for the other source, has a similar form. We compare the estimation error covariance (Eq. 13) with the corresponding CRB (Eq. 18). Both approaches attain the CRB with expected asymptotic performance, but the RMSE reveals that the SDF approach performs much better than the traditional TMA approach. During the path of the first moving sensor, the azimuth angle separation decreases. This geometry between sensor and sources leads to biased bearing estimates, to resolution conflicts and to mistakes by solving the bearing data association

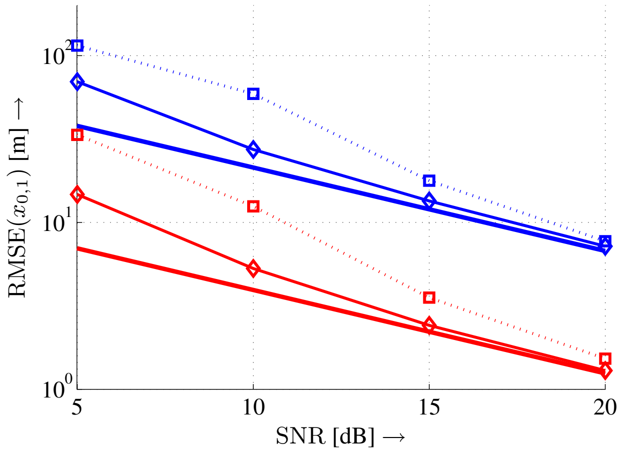

Figure 5: Square-root of the CRB (dashed lines) and the estimated RMSE for the traditional (dotted lines) and direct (solid diamond lines) TMA approach versus SNR for $x_{0,1}$-coordinate; only sensor 1 (blue), both sensors (red)

problem (for a non-ideal data association). Since the SDF approach does not compute intermediate (and maybe biased) bearings, this approach has a smaller RMSE. Furthermore the performance can be improved by using the additional second sensor.

5 Conclusions

We have extended the recently developed SDF approach to multiple sensors. We compared the direct SDF approach with the traditional TMA approach under the most favorable assumptions for the traditional approach: ideal M2T and T2T association and fully known DOA variances.

The following advantages of the SDF approach over the traditional TMA approach were reported:

1. The M2T and T2T association problem is completely circumvented and all source states are estimated in a single step.

2. Joint processing of all sensor data (as done by the SDF approach) provides enhanced performance in estimating the states of multiple sources. The SDF approach performs *always* better than or the same as the traditional approach. The SDF estimator attains the CRB in cases where the traditional approach does not reach the CRB.

3. For low SNR, the SDF estimator benefits from a full integration gain. Consequently, the SDF approach offers a tactical advantage of increasing the operational range or for locating weak sources.

4. The bias inherent in DOA estimates, which reduces the performance of the traditional approach, does not influence the direct state estimation where no intermediate parameters like DOAs are used. Furthermore, no additional parameters like DOA variances are required. The traditional approach decreases in performance for a mismatch of the DOA variances.

5. Correlations between DOAs of different sources, in particular for closely-spaced sources, that are not included in traditional TMA approach are completely considered in the SDF approach.

6. The state estimation accuracy can be improved by an additional sensor, even in the case that this sensor does not satisfy the observability condition.

Finally, we remark that the SDF approach can be extended to non-linear target movements and also non-identical array sensors.

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