Autonomous Tracking of Space Objects with the FGAN Tracking and Imaging Radar*

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Abstract: This paper presents the first progress made at FGAN/FHR-RWA towards the development of a robust autonomous method for the tracking of space objects with the Tracking and Imaging Radar (TIRA). For the acquisition phase an algorithm based on a Least-Squares Estimation of the state vector and *f and g series* will be presented and tested. For the tracking phase the suitability of the Extended Kalman Filter in mixed coordinates (EKF) and of the Piecewise-Constant Acceleration Converted Measurements Kalman Filter (PCA-CMKF) will be evaluated.

1 Motivation

The FGAN Tracking and Imaging Radar facility (TIRA, see Fig. 1) located near Bonn is a large-scale radar system for research, development, and experimental verification of techniques for non-cooperative space object reconnaissance. Some important parameters of the TIRA system are given in Table 1. TIRA consists of three major subsystems [Meh96]: a *34 m parabolic antenna* on a fully steerable computer controlled azimuth-elevation mount, a high power 4-horn monopulse L-band *tracking radar* with a detection sensitivity of 2 cm at 1,000 km range, and a wideband Ku-band *imaging radar* for the generation of ISAR images with a resolution of 6.3 cm at best. The very small 3 dB beam width (0.031°) of the Ku-band imaging radar without tracking capability requires high precision guidance by the coaxially mounted L-band tracking radar.

For precise orbit determination and imaging of space objects with a single ground-based sensor highly accurate and continuous (as long as possible) tracking is indispensable. Radar sensors are especially suitable for monitoring objects in Low Earth Orbits (LEO, below 2,000 km). However, tracking space objects ($v \approx 8$ km/s) by a large-scale mechanically steerable radar antenna places a great demand on the dynamics of the antenna driving system in order to avoid target loss. The antenna driving system of TIRA (about 240 metric tons movable mass) allows a maximum angular velocity of 24°/s (6°/s) and a maximum angular acceleration of $6°/s^2$ ($1.5°/s^2$) in azimuth (elevation).

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The current tracking concept of TIRA requires external information on the orbit of the space object for acquisition as well as for tracking. Up to now the U.S. Space Surveillance Network (SSN) still provides this information in form of orbital elements (Two-Line Elements, TLE) in adequate quality and free of charge for currently about 10,000 non-classified objects. It is feared however that this orbital information will no longer be provided or reduced in quality due to restrictive U.S. security policy. As a result of the rapidly growing number of space objects the frequency of mix-ups and loss of objects increases. Orbital elements of classified - usually military - space objects are not provided. Objects smaller than about 10 cm (e.g. space debris) are not catalogued. For temporary ballistic objects nominal trajectories are available at best.
Therefore, a new tracking concept for the TIRA L-band radar is investigated which should allow autonomous (i.e. without any external orbit information) tracking of space objects in the future.

Figure 1: TIRA (photomontage).

Table 1: Parameters of TIRA.

Parameter	**Tracking radar**	**Imaging radar**
Centre frequency	1.33 GHz	16.7 GHz
Bandwidth	250 kHz	2.1 GHz
Antenna gain	49.7 dB	73.2 dB
3 dB beam width	0.49^o	0.031^o
Peak power	1.5 MW	13 kW
Pulse length	1 ms	256 μs
PRF	< 40 Hz	< 400 Hz

2 Concept for Autonomous Tracking of Space Objects

The concept for autonomous tracking of highly dynamic space objects with TIRA consists of two phases: the acquisition and the tracking phase.

For the acquisition phase a *stare-and-chase* strategy is proposed: while the antenna illuminates a fixed observation volume (*stare*) a realtime algorithm is used to detect if an object is crossing this volume. The measurements obtained when the object was in the observation volume are processed to predict future state vectors of the object. Thus suitable guidance information can be provided to guide the antenna until the object once again enters the observation volume (*chase*). The critical point is the necessity of accurate guidance information during the relatively long time necessary for pointing the large antenna to the predicted object trajectory.

After successfully completing the acquisition phase, the tracking phase can be initialised, i.e. now the guidance information can be estimated from the new collected measurements by an additional tracking filter. The high non-linearity of the measurement and dynamic models provides the major difficulty in finding the most suitable tracking filter. For real-time application a trade-off between processing speed and tracking filter complexity has to be made.

2.1 Algorithm for the Acquisition Phase

Data collected in stare mode has special characteristics like very limited number of echoes (passage through observation volume takes only 1-3 s), poor range accuracy (unmodulated pulses are used) and low SNR (especially for small objects or large ranges). The algorithm for state vector prediction from stare data can be described as follows:

1. Range/range rate fusion: Range measurements are "improved" by adopting the temporal evolution given by the range rate measurements.

2. Measurements conversion: Debiased conversion [LBS93] is applied to convert polar measurements (with modified range) to a Cartesian topocentric coordinate system. In this way, the influence of pseudo-accelerations can be reduced.

3. State vector WLS Estimation: The object state vector at the closest approach to the beam centre is estimated using the Weighted Least-Squares method. Kepler motion is assumed during the passage. Weights are given by the converted measurement covariance matrix.

4. Prediction: Future object state vectors are predicted using *f and g series* [Esc65] and taking the effect of the J_2 perturbation into account.

2.2 Algorithms for the Tracking Phase

A sufficiently accurate dynamic model for tracking LEO objects in ECEF coordinates is given in (1), where r, R_E and ω_E are the position vector modulus, the Earth's equatorial radius and the Earth's angular velocity, respectively. The measurement model corresponds to the polar/Cartesian coordinates relationship [LJ01].

$$\frac{\partial}{\partial t}\begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \frac{-\mu}{r^3}\cdot\left(1+\frac{3\cdot J_2\cdot R_E^2}{2\cdot r^2}\cdot\left(1-\frac{5\cdot z^2}{r^2}\right)\right)\cdot x + 2\cdot\omega_E\cdot v_y + x\cdot\omega_E^2 \\ \frac{-\mu}{r^3}\cdot\left(1+\frac{3\cdot J_2\cdot R_E^2}{2\cdot r^2}\cdot\left(1-\frac{5\cdot z^2}{r^2}\right)\right)\cdot y - 2\cdot\omega_E\cdot v_x + y\cdot\omega_E^2 \\ \frac{-\mu}{r^3}\cdot\left(1+\frac{3\cdot J_2\cdot R_E^2}{2\cdot r^2}\cdot\left(3-\frac{5\cdot z^2}{r^2}\right)\right)\cdot z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ a_x \\ a_y \\ a_z \end{bmatrix} \quad (1)$$

Among several non-linear filtering approaches, two strategies with low computational effort based on Kalman filtering are considered here:

Extended Kalman Filter in Mixed Coordinates (EKF): The EKF applies Kalman filter framework to non-linear Gaussian systems, by first linearising measurement and dynamic models using a first-order truncated Taylor series expansion around the current estimates. In spite of the tedious calculation of the models' Jacobian matrices, EKF has become a widespread algorithm for nonlinear filtering because of its low computational effort and satisfying performance. The implementation of the EKF in mixed coordinates means that the prediction step takes place in Cartesian coordinates (like in the dynamic model), whereas the filtering step takes place in polar coordinates (like in the measurements).

Piecewise-Constant Acceleration Converted Measurements Kalman Filter (PCA-CMKF): A different approach consists of using converted measurements and a piecewise-constant acceleration model for linearisation of measurement and dynamic models, respectively. Thereby the filtering problem becomes linear and its optimal solution is the Standard Kalman Filter. With the PCA model the accelerations are calculated analytically using (1) and the current state vector estimates. In (2) the state vector prediction in coordinate x (equally for y and z) using the PCA dynamic model is shown.

$$\begin{bmatrix} x(t_{k+1}) \\ v_x(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 1 & t_{k+1} - t_k \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x(t_k) \\ v_x(t_k) \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \cdot a_x(t_k|t_k) \cdot (t_{k+1} - t_k)^2 \\ a_x(t_k|t_k) \cdot (t_{k+1} - t_k) \end{bmatrix} \quad (2)$$

3 Outcome of First Tests

Stare and subsequent tracking data from the same passage of different known space objects were collected using the current tracking concept of TIRA to test the proposed algorithms. It was observed that the acquisition algorithm allows an accurate state vector prediction from real stare data during at least *25 s*. This time interval would be sufficient to successfully complete the acquisition phase, since TIRA can reach any pointing direction in less than *20 s*. Three examples for different geometries and objects are shown in Table 2, where $T_{prediction}$ is the time interval during which the predicted state is inside the observation volume, i.e. prediction errors in elevation (EL) and traverse (TR) have to be smaller than 0.25°, in range (R) smaller than 10 km and in range rate (RR) smaller than 65 m/s. An orbit fit using the complete tracking data set was taken as a reference for error calculation.

Table 2: Prediction accuracy of the acquisition algorithm.

Object	EL at stare pos.	Range at stare pos.	$T_{prediction}$			
			EL	TR	R	RR
5 cm sphere	8°	1450 km	59.6 s	30 s	120 s	64.8 s
3 m satellite	48°	1050 km	29.3 s	129 s	74 s	27 s
15 cm satellite	74°	780 km	26.2 s	145 s	28 s	237 s

Tracking data collected after the acquisition phase were used to evaluate the performance of the proposed filters. Table 3 shows the root mean square error (RMSE) and the required

Table 3: PCA-CMKF vs. EKF.

Tracking filter	EKF	PCA-CMKF
RMSE	194 m	182 m
CPU time	1.6 ms	1.3 ms

mean processing time per iteration of both filters using tracking data of 15 different stare-and-tracking experiments. The tracking residuals for a 10 cm sphere using the tuned PCA-

CMKF are shown in Fig. 2. It is observed that the tracking accuracy is sufficient to keep the target within the L-band tracking radar beam during the whole passage and even within the very small Ku-band imaging radar beam for a considerable time interval.

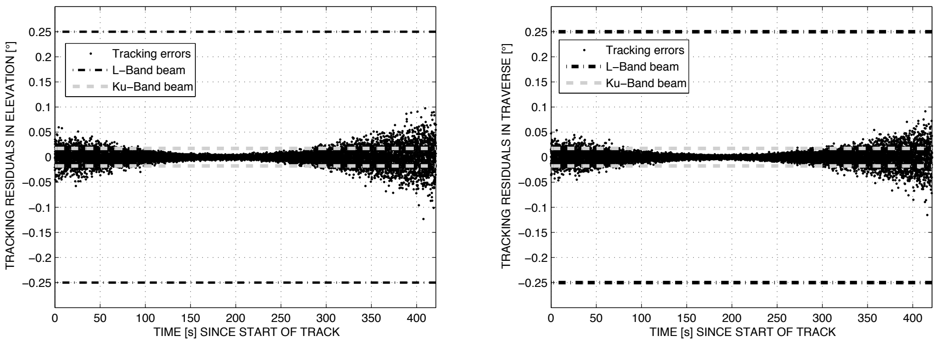

Figure 2: PCA-CMKF tracking residuals for a 10 cm sphere in elevation (left) and traverse (right).

4 Conclusions and Future Work

In this paper a robust and sufficiently accurate algorithm for the acquisition phase of an autonomous tracking method for space objects has been presented. It also has been demonstrated that for the tracking phase the PCA-CMKF can outperform the EKF and moreover requires 20% less computational effort. Although the performance of the presented algorithms is quite satisfying, the possibility of improvements based on more complex tuning techniques (e.g. Dynamic Model Compensation) and modern nonlinear filtering techniques (like the Unscented Kalman Filter or Particle Filters) will be studied.

References

[Esc65] P. R. Escobal. *Methods of Orbit Determination*. John Wiley & Sons, New York, 1965.

[LBS93] D. Lerro and Y. Bar-Shalom. Tracking with debiased consistent converted measurements versus EKF. *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 29, No. 3:1015–1022, July 1993.

[LJ01] X. R. Li and V. P. Jilkov. A survey of maneuvering target tracking–part II: Ballistic target models and part III: Measurement models. In *Proc. of SPIE Conf. Signal and Data Processing of Small Targets*, volume 4473, pages 559–581, 423–446, July-August 2001.

[Meh96] D. Mehrholz. Ein Verfolgungs- und Abbildungsradarsystem zur Beobachtung von Weltraumobjekten. *Frequenz*, Bd. 50, Ausg. 7-8:138–146, Juli-August 1996.