

# Multi-Target Tracking using Signal Strength Measurements

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**Abstract:** Many surveillance tasks rely on the generation of stable, continuous tracks of objects of interest. Often, track continuity has a higher priority than track accuracy, as valuable information on the identity or the origin of an object is lost by each track drop. The main source of track fragmentation are missing detections due to a limited field of view or technical or topographical masking. In the multi-target case, in addition, individual tracks can be interchanged by unknown data assignments. Therefore, the exploitation of additional sensor data is required in order to discriminate individual objects and to associate track fragments. As signal strength measurements are standard output of modern radar systems, the integration of this information into a Bayesian tracking scheme is discussed in the present paper. In contrast to previous approaches, the knowledge on the target's signal strength is not only used for an improved calculation of the association probabilities, but it enters into the algorithm as a random variable which is estimated sequentially. By this approach it is not only possible to discriminate closely-spaced targets and improve the track continuity, but also to support possible classification and identification tasks. The signal strength fluctuations of the target returns are modeled by the Swerling-I and Swerling-III cases. As a first performance evaluation, numerical results are presented based on a two-target simulation scenario.

## 1 Introduction

In ground surveillance with airborne Ground Moving Target Indication (GMTI) radar, the main task of establishing and maintaining tracks of relevant moving objects is challenged not only by imprecise, uncertain, and ambiguous measurements. To a large degree, the main difficulty arises from complex target dynamics, e.g. stop & go behavior or strong maneuvers, masking due to the sensors Doppler blind zone, nontrivial topography causing terrain obscuration, situations with closely-spaced targets, a strong false alarm background, etc. In general, these factors quickly lead to a strong performance degradation or even track loss. To counterbalance these factors, it is beneficial to incorporate additional sources of information into the tracking process.

Being a standard output of a modern radar system, the target amplitude or the corresponding target signal strength has already been used in the past to facilitate the problem of associating a track with its correct measurement. But in contrast to previous approaches, e.g. [LBS90, vK96, DCV10], in this work the knowledge on the target's signal strength is not only used for an improved calculation of the association probabilities, but it enters into

the algorithm as a random variable which is estimated sequentially. With this approach it is not only possible to discriminate closely-spaced targets and improve the track continuity, but also to support possible subsequent classification and identification tasks. In addition, for phased array antennas the signal strength estimates could also be used to contribute to the radar resource management: depending on the estimated target SNR, the detection threshold could be adapted to allow better target detections.

## 2 Signal strength model

Due to its complexity, a realistic modeling of a real target's back-scattering characteristics is in general impossible. Therefore statistical models are used instead which can be handled analytically. In this work it is assumed that the fluctuations of the input signal at the detector, resulting from fluctuations of the target cross section, can be described by the Swerling models [Swe60].

The complex target signal  $\mathbf{v} = (v_1, v_2)$  with orthogonal and statistically independent components is added by white Gaussian noise within the receiver unit. The detector uses the total signal  $\mathbf{u} = (u_1, u_2)$  to form the signal strength  $\|\mathbf{u}\|^2 = (u_1)^2 + (u_2)^2$  with the probability density given by the Rice distribution. In connection with the above mentioned fluctuation model and a detection threshold  $\lambda$ , this leads to explicit equations for the detection probability  $\mathbf{P}_D$  and the detected signal probability density  $p(s)$  as functions of target strength and  $\lambda$ . The functions  $p(s)$  are given in terms of Gamma functions (see [Koc06]) and will play the role of the likelihood function in the Bayesian update below.

## 3 Incorporation of signal strength information

In Bayesian target tracking, e.g. [BSL93, BSLK01], the probability density  $p(\mathbf{x}_k | \mathcal{Z}^k)$ , which describes the target state  $\mathbf{x}_k$  at time step  $t_k$ , conditioned on the measurement sequence  $\mathcal{Z}^k = \{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k\}$  with  $\mathbf{z}_k = \{z_k^i\}_{i=1}^{n_k}$ , is sequentially updated by

$$p(\mathbf{x}_k | \mathcal{Z}^k) = \frac{p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathcal{Z}^{k-1})}{\int d\mathbf{x}_k p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathcal{Z}^{k-1})} \quad (1)$$

To incorporate signal strength, the kinematic target state  $\mathbf{x}_k$  is augmented with a target strength variable  $s_k$  and, thus, becomes  $\mathbf{X}_k = (\mathbf{x}_k, s_k)$ . The signal strength measurements  $\kappa_k^i$ , on the other hand, are introduced into the measurement set:  $\mathbf{Z}_k = \{z_k^i, \kappa_k^i\}_{i=1}^{n_k}$ . We assume that the target density factorizes in  $\mathbf{x}_k$  and  $s_k$ . As usual in the tracking literature, the density on  $\mathbf{x}_k$  is described by normal distributions, predicted and updated, for given target–detection assignments, with the Kalman filter. Because of the structure of the likelihood function, the density on  $s_k$  is well described by an inverse Gamma function

which keep its structure under successive application of the Bayes update:

$$p(\mathbf{s}_k | \mathcal{Z}^{k-1}) = \mathcal{I}(\mathbf{s}_k; \hat{\mathbf{s}}_{k-1}, \mu_{k-1}) = N_{\mu_{k-1}} \mathbf{s}_k^{-\mu_{k-1}-1} e^{-\frac{(\mu_{k-1}-1)\hat{\mathbf{s}}_{k-1}}{\mathbf{s}_k}} \quad (2)$$

with a normalization constant  $N_{\mu_{k-1}}$ . The probability density  $\mathcal{I}(\mathbf{s}; \hat{\mathbf{s}}, \mu)$  has the expectation value  $E[\mathbf{s}] = \hat{\mathbf{s}}$ . If the parameter  $\mu > 2$ , then the variance exists with  $\text{Var}[\mathbf{s}] = \frac{\hat{\mathbf{s}}^2}{\mu-2}$ . Because of  $\frac{d}{dt} \text{SNR}_0 = 0$ , no “dynamics” is needed for the signal strength, thus the prior density at  $t_k$  is identical to the posterior density at  $t_{k-1}$ .

### 3.1 Combined likelihood

The combined likelihood function  $p(\mathbf{Z}_k | \mathbf{X}_k) = p(\mathbf{Z}_k | \mathbf{x}_k, \mathbf{s}_k)$  is the probability density of the measurements and comprises all possibilities how the given sensor output  $\mathbf{Z}_k$  can be interpreted, given the true target state  $\mathbf{X}_k$ . Assuming independent, identically distributed false alarm measurements with the number of false alarms determined by the Poisson distribution, the likelihood function, up to a factor which does not depend on  $\mathbf{X}_k$ , can be written as

$$p(\mathbf{Z}_k | \mathbf{X}_k) \propto (1 - \mathbf{P}_D(\mathbf{s}_k)) \rho_F + \mathbf{P}_D(\mathbf{s}_k) \sum_{i=1}^{n_k} \mathcal{N}(z_k^i; \mathbf{H}_k \mathbf{x}_k, \mathbf{R}_k) \mathcal{LR}_S^i \quad (3)$$

where  $\rho_F$  is the false alarm density and  $\mathcal{N}(z_k^i; \mathbf{H}_k \mathbf{x}_k, \mathbf{R}_k)$  is the normally distributed single measurement likelihood, which results from a linear measurement model with additive white Gaussian noise:  $z_k^i = \mathbf{H}_k \mathbf{x}_k + v_k$ ,  $\mathbf{v}_k \propto \mathcal{N}(\mathbf{v}_k; 0, \mathbf{R}_k)$  with measurement matrix  $\mathbf{H}_k$  and measurement covariance  $\mathbf{R}_k$ . The signal strength likelihood ratio  $\mathcal{LR}_S^i$  in (3) is given by the above mentioned signal densities  $p(s)$ .

### 3.2 Filter update step

Assuming strong targets, i.e.  $1 + \mathbf{s}_k \approx 2 + \mathbf{s}_k \approx \mathbf{s}_k$ , the posterior density  $p(\mathbf{X}_k | \mathcal{Z}^k) = p(\mathbf{x}_k, \mathbf{s}_k | \mathcal{Z}^k)$  can be written as the sum of the product of a Gaussian with an inverse Gamma density, describing the target’s kinematic state and signal strength, respectively:

$$p(\mathbf{X}_k | \mathcal{Z}^k) = \sum_{i=0}^{n_k} w_k^i \mathcal{N}(\mathbf{x}_k; \mathbf{x}_{k|k}^i, \mathbf{P}_{k|k}^i) \mathcal{I}(\mathbf{s}_k; \mathbf{s}_{k|k}^i, \mu_{k|k}^i). \quad (4)$$

where the estimates  $\mathbf{x}_{k|k}^i$  and covariances  $\mathbf{P}_{k|k}^i$  are calculated by the known Kalman filter update equations, and weights  $w_k^i$ , signal strength estimates  $\mathbf{s}_{k|k}^i$  and  $\mu_{k|k}^i$  are given in [Koc06].

### 3.3 Algorithmic implementation

The presented update scheme is implemented into the single-target PDAF [BSL93] and the multi-target JPDAF [BSLK01] tracking algorithms. In the PDAF algorithm,  $n_k$  measurements lead to  $n_k + 1$  hypotheses which are merged to a single main hypothesis by second-order moment matching at the end of the filter update step. In the JPDAF algorithm, the complete set of possible global hypotheses is processed, i.e. all possible combinations to associate measurements to tracks including missed detections and false alarms are considered. Thus it avoids the association of a certain measurement to more than one track, which is a shortcoming of the simple single-target PDAF.

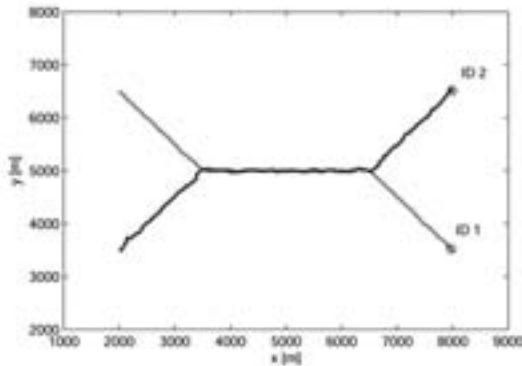


Figure 1: Two-target simulation scenario. Shown is the snapshot at the final revisit time.

## 4 Simulation scenario & results

We consider a simulation with two targets approaching and moving along the same trajectory for a considerable period of time (see Fig. 1), leading to a loss of identity when traditional tracking algorithms are used. The objective is to determine the capability of target discrimination with signal strength information in the final stage of the scenario. Based on the JPDAF algorithm and the simulation parameters given in Tab. 1, the results are plotted in Fig. 2 for Swerling-I and III fluctuations, respectively, of the two targets. The figure depicts the probability for correct (solid lines) and incorrect (dashed lines) association of tracks with true targets at the final revisit for different signal strength combinations, based on the two fluctuation models. The signal strength of one target is fixed, corresponding to the minimum value of each line. The upper plots correspond to target tracking without signal strength information. In this case, the signal strength only affects the true detection probability and thus the occurrence of target detections. As expected, for all combinations of the two targets' signal strength the probability for correct association amounts to 50%. The lower plots illustrate the advantage of taking signal strength information into account:

Table 1: Simulation parameters

Monte Carlo runs	$N_{MC}$	=	1000
Target velocity	$v_{target}$	=	15 m/s
Process noise	$\sigma_p$	=	0.5 m/s <sup>2</sup>
Range error	$\sigma_r$	=	10 m
Azimuth error	$\sigma_\varphi$	=	0.25
Mean false alarms	$\bar{n}_{FA}$	=	1
Field of view	$ V $	=	10 km $\times$ 10 km
Sensor position	$\vec{r}_{sensor}$	=	$[-1, 5, 10]^T$ km
Revisit rate	$\Delta_T$	=	2 s
Detector threshold	$\lambda$	=	4
Mean clutter SNR	CNR <sub>0</sub>	=	10

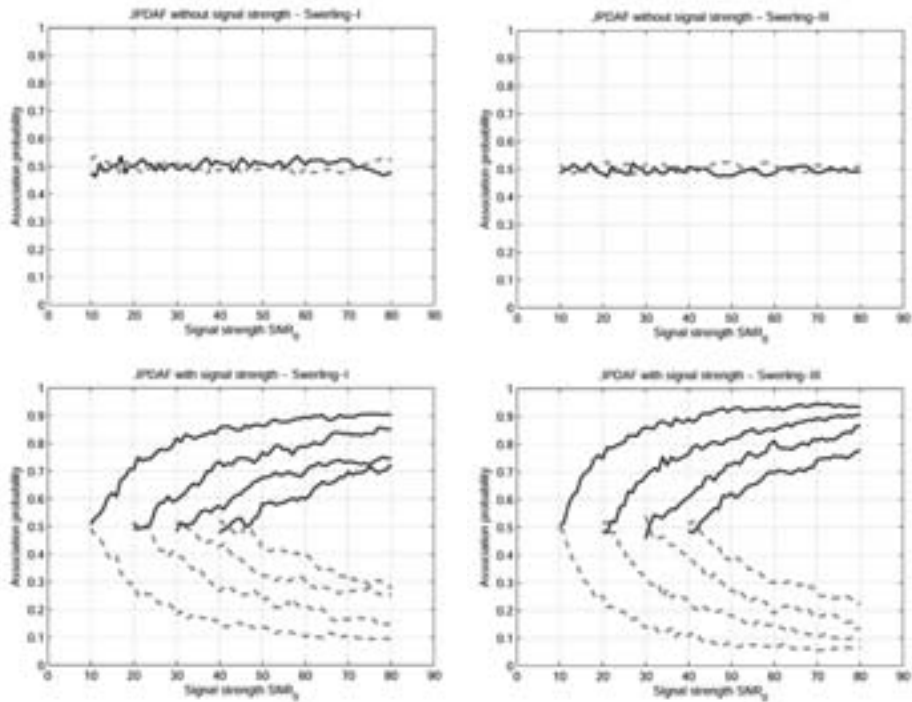


Figure 2: Probability for correct (solid lines) and incorrect (dashed lines) association of tracks with true targets at final revisit for different signal strength combinations, based on fluctuation model Swerling-I (left) and Swerling-II (right). The signal strength of one target is fixed, corresponding to the minimum value of each line. Shown are the results for JPDAF without (above) and with (below) exploitation of signal strength information.

A clear performance gain in target discrimination is visible for both fluctuation models, with a stronger separation capability for larger differences in signal strength. Comparing the results in Fig. 2 (lower plots), apparently the discrimination performance of the algorithm is slightly higher in case of Swerling-III. This is probably due to the distinct peak in the probability density of Swerling-III fluctuations, leading to a more obvious discrimination of the two signal strength distributions.

## 5 Conclusions

In this paper we developed a tracking algorithm which incorporates signal strength information. In contrast to previous approaches, the knowledge on the target's signal strength is not only used for an improved calculation of the association probabilities, but it enters into the algorithm as a random variable which is sequentially estimated. Based on simulation scenarios, the performance of the presented algorithm is evaluated. As expected, the exploitation of signal strength leads to a performance gain in target discrimination in a multi-target scenario.

Further work will focus on the incorporation of signal strength information into the CPHD algorithm [Mah07] and on a multiple model approach for different target fluctuation models.

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