On the modification of phonon tracing

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Abstract: Phonon tracing is a geometric approach estimating acoustics in closed rooms. This work contains discussions of possible refinements and extensions of the algorithm.
In computer graphics exist numerous level-of-detail approaches decreasing the representation details of objects in order to speed up computations and rendering of virtual scenes. Different ideas for the realization of a level-of-detail approach in acoustics are presented. For this purpose the phonon tracing algorithm has to be modified. With these modifications the room impulse response can be calculated with respect to the user requirements on calculation time and accuracy.

1 Introduction

Most methods applied for acoustic simulation of enclosures are based on geometric acoustics [AB79, Bor84, Kro68, Kul84, Vor89, FCE$^+$98, KJM04, Sve02]. Prevalent, the sound energy or intensity transported from the sound source to the listener is determined. In our previous work [BDM$^+$05] we presented the phonon tracing algorithm, an energy based approach for room acoustic simulation. The present paper covers several modifications and extensions of the phonon tracing algorithm.

In contrast to sound pressure the sound energy quantities can hold only positive values. Thus modeling of interference phenomena is not possible. Therefore we modify the phonon tracing algorithm such that it traces pressure instead of energy facilitating the representation of interference.

If the structure of a surface is of the same order as the wave length, diffuse reflection of the sound wave front occurs. We extend the tracing step of the phonon tracing algorithm considering diffuse reflection depending on given scattering coefficients.

Furthermore we describe a modification of the collection step of the phonon tracing algorithm in order to allow an adaptive collection of the sound particles. The idea is to adjust the level of detail of the calculated room impulse response to the user requirements on the calculation accuracy and time. Therefore, we subdivide the particles stored in the phonon map into clusters of similar trajectories. Depending on the user's input parameters a subset of the phonons is collected determining the impulse response.

The remainder of our paper is structured as follows. In the next section we will describe the modified phonon tracing algorithm which calculates the pressure response instead of energy. In section 3 we describe the integration of diffuse reflection into the tracing step. Afterwards, in section 4 we present a novel level of detail approach to phonon tracing. Finally we will conclude our paper with a discussion of future work.

2 Tracing the sound pressure

This section describes the modification of the phonon tracing approach, where sound pressure is used for calculations, instead of energy. The main idea is, analogous to the algorithm described in [BDM+05], to trace sound particles outgoing from the sound source through the given scene constructing the phonon map. Afterwards the phonons are collected in order to calculate the room impulse response at a given listener position. In contrast to the energy approach where quadratic attenuation is used because of the spatial particle density, linear pressure attenuation is facilitated. This is obtained by modeling linear pressure attenuation by Gaussian basis functions dilated proportional to the traversed distance (see figure 1). Here, single particles can be considered as individual micro-sources in analogy to the image-source method.

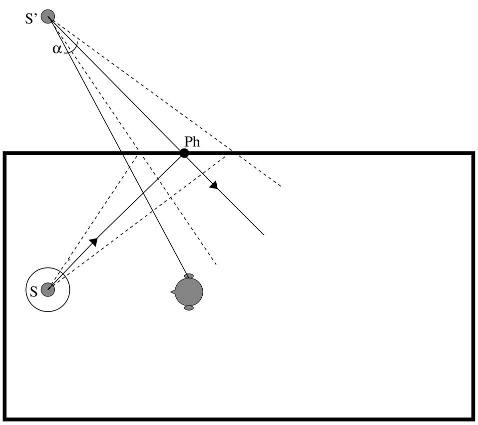

Figure 1: Sketch of the phonon tracing algorithm to trace sound pressure inside closed rooms.

2.1 Algorithm specification

We define:

- S a spherical sound source in point x_0 in $\mathbb{R}^3$
- S_s a unit sphere in $\mathbb{R}^3$, i.e. $S_s = \{x \in \mathbb{R}^3; \|x\| = 1\}$

- $\{v_i\}_{i=1,...,N}$ a set of directions distributed on S_s with regard to the emission distribution of the sound source S
- $\psi_i : S_s \mapsto \mathbb{R}, x \mapsto g(\arccos(\langle x, v_i \rangle)), i = 1, ..., N$ the partition of unity over S_s

For a unit impulse (Dirac) δ at time delay $\Delta t = \frac{1}{f_s}$ the pressure $P(t, x)$ at time $t \in \mathbb{R}$ in x reads:

$$P(x,t) = P_1 \frac{1}{\|x\|} \delta \left(t - \frac{\|x\|}{c} \right) \sum_{i=1}^{N} \psi_i \left(\frac{x}{\|x\|} \right) \tag{1}$$

where P_1 is the pressure in 1 m distance from the sound source S, c is the sound velocity, f_s the sampling rate, and $\delta(t)$:

$$\delta(t) = \begin{cases} 1 & \text{if } t \in [0, \Delta t] \\ 0 & \text{else} \end{cases} \tag{2}$$

As mentioned before we choose g as Gaussian basis function of an angle $\vartheta = \arccos(\langle x, v_i \rangle)$, $i = 1, ..., N$:

$$g(\vartheta) = \alpha \cdot \exp \left(-\frac{\vartheta^2}{2\sigma^2} \right) \tag{3}$$

For evaluation of the room acoustics a uniform distribution of sound rays is used. In order to achieve the partition of unity of the basis functions for the uniform distribution (the needed parameter can also be derived for non uniform distributed direction vectors), it must hold:

$$\int_{S_s} g \, \mathrm{d}S = \frac{4\pi}{N} \Leftrightarrow \alpha = \frac{2}{N} \left(\int_{\vartheta=0}^{\pi} \exp \left(-\frac{\vartheta^2}{2\sigma^2} \right) \cdot \sin\vartheta \, \mathrm{d}\vartheta \right)^{-1} \tag{4}$$

where 4π is the surface area of the unit sphere S and N the number of direction vectors. The integral in the above equation has to be solved numerically, but if we assume $\sigma \ll 1$, then it holds:

$$\int_{\vartheta=0}^{\pi} \exp \left(-\frac{\vartheta^2}{2\sigma^2} \right) \cdot \sin(\vartheta) \, \mathrm{d}\vartheta \approx \int_{\vartheta=0}^{\pi} \exp \left(-\frac{\vartheta^2}{2\sigma^2} \right) \mathrm{d}\vartheta \tag{5}$$

By substituting $u = -\frac{\vartheta^2}{2\sigma^2}$ and $\mathrm{d}u = -\frac{1}{\sigma^2} \vartheta \, \mathrm{d}\vartheta$ we get:

$$\int_{\vartheta=0}^{\pi} \exp \left(-\frac{\vartheta^2}{2\sigma^2} \right) \mathrm{d}\vartheta \approx \sigma^2 \tag{6}$$

This leads to:

$$\alpha = \frac{2}{N\sigma^2} \tag{7}$$

Choosing σ there is a trade-off between smoothness of the partition of unity and the resolution of geometric details of the scene. For better approximation of the partition of unity the support of the Gaussian function g should contain n different direction vectors. The surface area for a direction reads:

$$a = n\frac{4\pi}{N} \tag{8}$$

Furthermore to fulfill the partition of unity the following must be true for an angle ϑ_0:

$$\exp\left(-\frac{\vartheta_0^2}{2\sigma^2}\right) \geqslant \frac{1}{2} \;\Rightarrow\; \sigma = \frac{\arccos\left(1 - \frac{2n}{N}\right)}{\sqrt{2\ln 2}} \tag{9}$$

Thus, the Gaussian basis function with an approximated partition of unity over the unit sphere S reads:

$$g(\vartheta) = \frac{2}{N\sigma^2} \cdot \exp\left(-\frac{\vartheta^2}{2\sigma^2}\right) \tag{10}$$

Halton sequence. For the accuracy of the Monte-Carlo integration, the uniformity is more important than the randomness. The Halton quasi-random sequence (also applied in computer graphics [Kel96a, Kel96b]) is used for generating direction vectors. The Halton sequence is obtained by calculating the radical inverse function, which takes a number $i \in \mathbb{N}$ represented in prime base p and reflects it through the radical point:

$$\Phi_p(i) := \sum_{j=0}^{\infty} a_j(i)p^{-j-1} \in [0,1) \Leftrightarrow i = \sum_{j=0}^{\infty} a_j(i)p^j \tag{11}$$

The m dimensional Halton sequence reads:

$$z_i = (\Phi_{p_1}(i), ..., \Phi_{p_m}(i)) \tag{12}$$

The radical inverse function can be intutively understood in the following way. To get the radical inverse of i in base p, we write down the digits of i in the base p, then we invert their order and place the floating point in the beginning. The resulting number is a desired floating point number, expressed in base p. The Halton sequence is incremental, i.e. it is possible to increase the number of samples without discarding the already calculated samples. The algorithm described in [HS64] can be used for the calculation of the Halton sequence, which is not more expensive as determination of usual pseudo-random numbers.

2.2 Implementation

Now, as we have defined the Gaussian basis functions we can formulate the phonon tracing algorithm to trace pressure. The algorithm requires the following input parameters:

- position of the sound source S
- reference pressure P_1 in 1 m distance from S
- emission distribution E of S
- one ore more listener positions l_i
- a triangulated scene with tagged materials m_j
- a reflection function $\rho_j : \Omega \mapsto [-1, 1]$ for each material
- an acoustic BRDF for each material (if applicable)
- a number of phonons n_{ph} to be traced from the source
- a lower pressure threshold ϵ and a maximum number of reflections n_{refl} for terminating the phonon paths

The output of our approach is a filter f_i for each listener position l_i corresponding to the impulse response with respect to the sound source and the phonon-map. For each phonon ph the map contains the pressure spectrum p_{ph}, the phonon's position pt_{ph} at the reflection point, the image source q_{ph} from which we can calculate the phonons outgoing direction v_{ph} and the traversed distance d_{ph}, number of reflections r_{ph}, and the material m_{ph} at the current reflection.

2.2.1 Phonon emission step

The phonons sent out from the source are associated with the following quantities:

- a pressure spectrum $p_{ph} : \Omega \mapsto \mathbb{R}^+$
- the virtual source q_{ph}
- the phonon's current position pt_{ph}

As described in [BDM$^+$05] $n_e = 10$ frequency bands are used for the simulation. Furthermore, the wavelets introduced in [BDM$^+$05] are the basis functions for the pressure spectrum.

Phonons are emitted from the source S according to the emission probability distribution E and have a unit pressure spectrum $p_{ph,i} = 1$ $(i = 1, ..., n_e)$ at their starting point. At the intersection of the phonon ray with the scene, the virtual source q_{ph} is calculated as follows:

$$q_{ph} \quad \leftarrow \quad q_{ph} + 2 \cdot \langle (pt_{ph} - q_{ph}), n \rangle \cdot n \tag{13}$$

where n is the surface normal at the intersection point pt_{ph}, $\langle \cdot, \cdot \rangle$ denotes the scalar product. The pressure is reduced according to the reflection function ρ_j of the local material m_j. The phonon is fixed at the intersection point, and stored in the global phonon map. If the termination conditions are not satisfied, the tracing process is continued. Otherwise a new phonon is started from the source. The flow chart in figure 2 outlines the emission step.

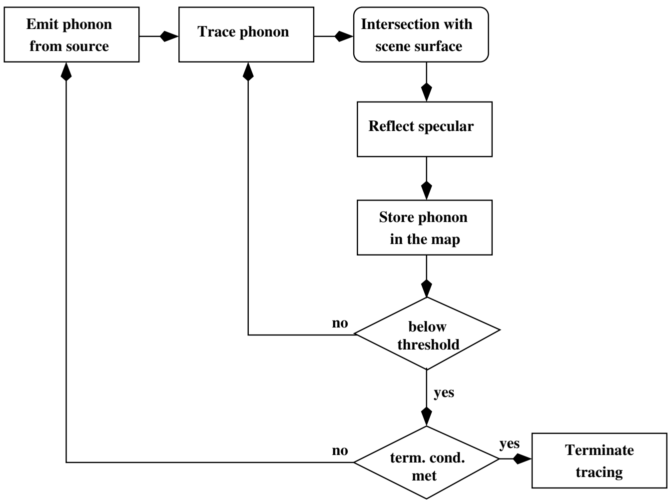

Figure 2: Flow chart diagram of the phonon emission step.

2.2.2 Phonon collection and filtering

The phonons are now collected in order to build the impulse response f_i at the listener position l_i. In the case of uniform absorption for all frequencies the contribution of a single phonon to the impulse response, corresponding to equation 1 reads:

$$p(t, x) = \frac{\rho_{tot} \, P_1}{d_{ph}} \, g\bigg(\angle \left(v_{ph}, l_i - q_{ph} \right) \bigg) \cdot \delta \bigg(t - \frac{d_{ph}}{c} \bigg) \tag{14}$$

where P_1 is the reference pressure at 1m from the source, ρ_{tot} is the product of the reflection coefficients along the phonon path and g is a Gaussian weighting function defined in equation 10 with $N = n_{ph}$.

Since most absorption coefficients α provided in the literature refer to energy, we must calculate the pressure related reflection coefficients:

$$\rho = \sqrt{1-\alpha}\,. \tag{15}$$

In the general case of frequency-dependent absorption, the unit pulse is subdivided as presented in [BDM$^+$05]. The filter f_i then becomes a sum of wavelets (band pass filters) scaled and shifted as described above. Furthermore, the air absorption is considered, too.

3 Modeling diffuse reflections

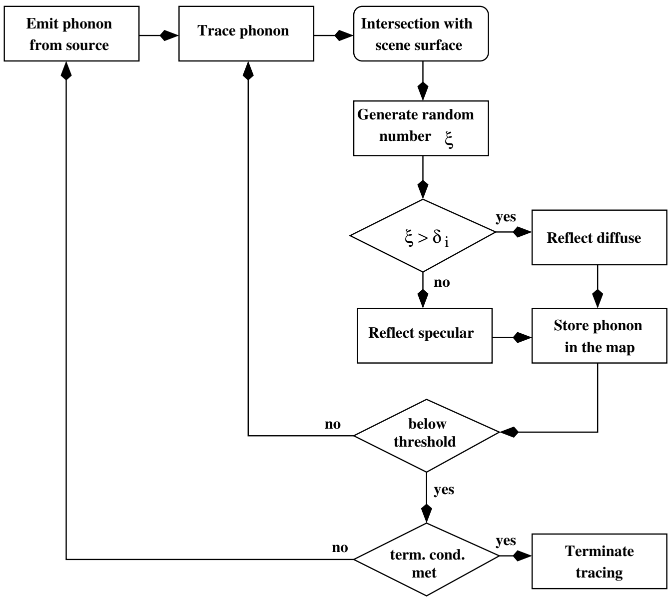

Figure 3: Flow chart diagram of the phonon emission step including diffuse reflections.

In the phonon tracing algorithm described in the previous section only specular reflections are considered. In fact for virtual acoustic applications surfaces are generally assumed to reflect the sound wave specularly. This assumption is true if the surface structure is significantly smaller in contrast to the wave length. Otherwise, the sound wave is reflected diffusely. This section describes the diffuse reflection model used in the modified phonon tracing algorithm.

In room acoustics, in order to model diffuse reflection, a scattering coefficient δ_i for each scene surface is introduced. The *phonon emission step* of the phonon tracing algorithm is then modified as follows. If the sound particle hits a scene surface, a number $\xi \in [0,1]$ is randomly chosen. If $\xi > \delta_i$ the phonon is reflected diffusely, otherwise the reflection is specular. In case of diffuse reflection the outgoing direction v_p is determined assuming ideal (Lambertian) diffuse reflection where the direction of the reflection is perfectly random over a hemisphere surrounding the phonon's position p_p (intersection point of the ray with the surface) [Kut71]. The azimuthal angle θ is a random number ξ_1 in the interval $(-\pi, \pi)$ and the polar angle ϕ is given by the arc-cosine of the square root of a random number ξ_2 chosen in the interval $(0, 1)$:

$$\theta = \xi_1 \qquad \phi = \arccos(\sqrt{\xi_2}) \tag{16}$$

In the pressure phonon tracing algorithm the virtual source in equation 13 can now be calculated as follows:

$$q_p = p_p - d_p \cdot v_p \tag{17}$$

where p_p is the phonon's current position and d_p the traversed distance. Figure 3 depicts the *phonon emission step* including the diffuse reflection modeling.

Since the scattering coefficients δ_i are frequency dependent, we have to either perform a tracing step for each frequency band or use an average scattering coefficient for all frequencies. A better solution for this situation is subject to future research.

4 Acoustic level of detail

In computer graphics, there exist multiple techniques considering the level of detail representation of geometric objects [LRC+03]. In Virtual Reality for example the objects located in larger distance from the viewer position are rendered using fewer patches reducing rendering time. More details are added by the time the viewer is getting closer to the object providing more accurate representation. This section describes some different ideas how the phonon tracing algorithm can be modified in order to provide calculation of the room impulse response in different levels of detail depending on user requirements on computation time and computation accuracy.

4.1 Clustering

Before we discuss the different level of detail approaches we need to subdivide the particles stored in the phonon map into clusters which represent different wave-fronts which are being reflected by the walls. We use the information stored in the phonon map to accomplish the subdivision process. For early reflections all phonons within a cluster have the same trajectory, that is they are being reflected at the same plane and have equal absorption properties. Thus they satisfy the following criteria:

- equal numbers of reflections n_p
- and for each reflection:
  - equal material indices m_p (same object of the scene)

– equal surface normals n at the reflection position

Consequently phonons included in an early reflection cluster have equal virtual source position and pressure absorption factor.

At higher reflection orders, the clusters become smaller and smaller, until they contain only a single phonon. Hence, for cluster reconstruction at higher reflection order we disregard the constraint of equal trajectory and combine phonons having equal last reflection material. The cluster criteria are then as follows:

- equal material index m_p of last reflection
- equal surface normal n at the last reflection position

These clusters contain the residual phonons for different room surfaces after a prescribed number of first reflections. All the phonons within such residual cluster have different virtual sources and also different pressure decompositions.

4.2 LoD approaches

In the following, different variants of a level of detail method using the cluster information will be discussed.

The idea of providing different levels of the impulse response can be summarized in the following steps:

1. Find a representative phonon ph_r for each cluster
2. Calculate distance d_i of each phonon in the cluster to ph_r
3. Sort phonons according to d_i
4. Collect only phonons which are in a user prescribed distance d from ph_r

Now we will describe these individual steps in more detail.

Step 1. In the first step we determine the position pt_{ph_r} of the representative phonon ph_r for each cluster consisting of n particles:

$$pt_{ph_r} = \frac{\sum_{i=1}^{n} w_i \cdot pt_{ph,i}}{\sum_{i=1}^{n} w_i} \tag{18}$$

The weight w_i used for a single phonon depends on the phonon's pressure absorption factor due to reflections at the room walls. Since all phonons in an early reflection cluster have the same absorbtion factor the weights are all equal to 1 ($w_i = 1$). Whereas inside the late reflection clusters the weight w_i grows inversely proportional to this factor. Thus the representative phonon ph_r is shifted towards phonons having a amount of pressure absorbed due to reflections at least in one frequency band.

Step 2. Now we calculate the distance d_i of each phonon $pt_{ph,i}$ inside the cluster as Euclidean distance to ph_r:

$$d_i = |pt_{ph,i} - pt_{ph_r}| \tag{19}$$

The distances are used for the adaptive collection of particles in the next steps.

Step 3. In order to ensure a rapid collection of particles we sort them in ascending order with respect to the distance d_i.

Step 4. The information precalculated in the previous steps is now used for the collection of the particles. The user can prescribe a distance from the representative phonon ph_r. All phonons which are not farther away than this distance are collected. Another possibility is to predefine the number m of phonons to collect. Then only the m nearest particles to ph_r in the cluster are considered for the impulse response calculation. Thus, for a lowest level of detail only the representative phonon of each cluster is collected and all particles for the finest detail calculation.

5 Summary and Discussion

In this paper we have presented several modifications of the phonon tracing algorithm for room acoustics. The first modification was the improvement of the algorithm in order to trace pressure instead of energy allowing modeling of interference phenomena. Furthermore we extended the tracing step of the algorithm to consider diffuse reflections. We also presented a possibility to provide different levels of detail of the calculated impulse response depending on user input.

When including diffuse reflections one problem comes up. Since the scattering coefficients are frequency dependent one tracing step for all considered frequency bands is not enough. We either have to apply an average scattering coefficient and perform one tracing step, or we have to trace the phonons for each frequency band separately.

Concerning the level of detail we have presented a possibility to adapt the collection step of the algorithm depending on the users requirements on the accuracy of the resulting impulse response. Therefore we subdivided the particles in the phonon map into clusters and collected only a subset of the entire particles depending on the distance of the phonons to a representative phonon of the cluster. Further validations and verifications of this approach are necessary. In room acoustics early reflections are of greater interest as late reverberations, thus an approach incorporating this could be more appropriate. Additional benefits regarding the clustering should be investigated, for example the reduction of memory use for phonon map storage. All phonons inside early reflection clusters have equal total reflection coefficient, thus this information need to be stored only once for the cluster and not for each phonon separately. Also, the position of the virtual source is equal, so we can store only the position of the virtual source and the pressure absorption coefficient for the cluster at the room surface. Statistical methods can then be used to perform the collection step, for example, collection of a prescribed number of particles equally distributed in the cluster. Therefore the regions of the clusters also have to be determined and stored. Regarding the residual particle clusters, more information has to be stored, due to the fact that the particles have both different pressure decomposition and different virtual source positions. Statistical approaches can be applied if the pressure as well as the virtual source positions distributions can be determined.

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