Reentry of Space Objects: Tracking and Classification with Sequential Monte Carlo Techniques

Felix Opitz

Defence and Communication Systems / Air and Naval Defence
EADS
Wörthstr. 85
89077 Ulm, Germany
felix.opitz@eads.com

Abstract: A new approach of tracking and classification of space objects with sequential Monte Carlo methods and numerical integration techniques is given.

1 Introduction

One application of multi sensor data fusion is the tracking of space objects via ground based sensor networks. Therefore, the space objects may be recognized through the boost, costal, reentry phases or during stationary orbits. This paper is restricted to the reentry of space objects, like satellites in decaying orbits, falling debris, reentry vehicles. The reentry dynamics is found to be highly nonlinear. Therefore, one has to combine recently founded non linear tracking techniques, so called sequential Monte Carlo methods with numerical integration. Finally, the trajectory depends on a priori unknown attributes of objects, which are also estimated and may be used to classify and discriminate targets.

2 Fundamental Astro- and Aerodynamics

2.1 Coordinate Transformations

The radar origin is the centre of the ENU coordinate system. The x-axis of the ENU system is directed to east, y-axis to north and z-axis upwards. The measurement system RUV is also centred in the phased array radar, given by the range and the direction via the two cosinus u and v w.r.t. to the antenna array. The transformation T_{ENU}^{RUV} is:

$$\begin{pmatrix} \vec{r}' \\ \vec{v}' \end{pmatrix} = \begin{pmatrix} T\vec{r} - \vec{R}_0{}' \\ T\vec{v} \end{pmatrix} \text{ with } T = \begin{pmatrix} \cos\lambda & -\sin\lambda & 0 \\ \cos\phi\sin\lambda & \cos\phi\cos\lambda & -\sin\phi \\ \sin\phi\sin\lambda & \sin\phi\cos\lambda & \cos\phi \end{pmatrix} \tag{1}$$

$$r = \sqrt{x'^2 + y'^2 + z'^2} \qquad u = \frac{x'}{r} \qquad v = \frac{y'}{r} \tag{2}$$

The aerodynamics of the object is best described in an object centred coordinate system VTC, defined by its velocity direction $\vec{e}_v$ and two directions $\vec{e}_t$ and $\vec{e}_c$, defined $\vec{e}_t = \vec{e}_z \times \vec{e}_v$ and $\vec{e}_c = \vec{e}_v \times \vec{e}_t$. The transformation from ENU into VTC is called T_{ENU}^{VTC}.

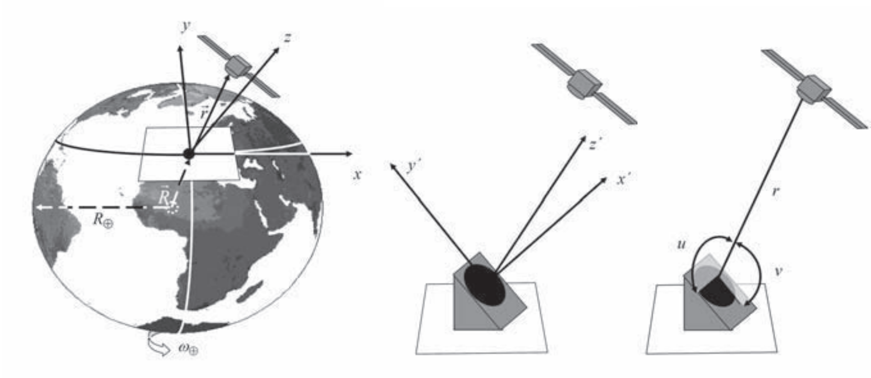

Figure 1: Coordinate systems for space applications.

In the ENU coordinate system the dynamics of a space object state $\vec{x} = (\vec{r}, \vec{v})^{\mathrm{T}}$, $\vec{r}$ resp. $\vec{v}$ the position resp. velocity, is given by the differential equation:

$$\frac{d\vec{x}}{dt} = \vec{f}(t, \vec{x}) = \left(\vec{v}, \quad \vec{a}(\vec{r}, \vec{v})\right)^{\mathrm{T}} \tag{3}$$

$$\vec{a}(\vec{r}, \vec{v}) = \underbrace{\vec{g}(\vec{p})}_{gravity} \underbrace{-2(\vec{\omega}_\oplus \times \vec{v})}_{coriolis} \underbrace{-\vec{\omega}_\oplus \times (\vec{\omega}_\oplus \times \vec{p})}_{centrifugal} + \underbrace{\vec{a}_A}_{aerodynamik} \tag{4}$$

Here, $\vec{p} = \vec{r} + \vec{R}$ is the object position w.r.t. to the Earth's centre, $\vec{R}$ the vector from the geocentre to sensor origin and $\vec{\omega}_\oplus$ the Earth's angular velocity. [BM71, Me71, LJ01]

2.2 Geopotential

The gravity for the elliptic model is given by

$$\vec{g}(\vec{p}) = -\frac{\mu}{p^2}\left(\vec{e}_p + \frac{3J_2 R_\oplus^2}{2p^2}\left(\vec{1} - 5(\vec{e}_p^{\mathrm{T}} \vec{e}_N)^2\right)\vec{e}_p + 2(\vec{e}_p^{\mathrm{T}} \vec{e}_N)\vec{e}_N\right) \tag{5}$$

with the constants $\mu = 3.986005 \cdot 10^{14} \frac{m^3}{s^2}$, $J_2 = 0.108627989 \cdot 10^{-2}$ the second zonal harmonic coefficient and $R_\oplus = 6378137m$ the Earth's equatorial radius. [Co94]

2.2 Aerodynamic Forces: Drag and Lift

Both, drag and lift forces, depend on the atmospheric density. There are many sophisticated models about the air density. An easy and widely applied approach is given through the following, high dependent model [Za97]:

$$\rho(h) = 1.225\exp(-\frac{h}{9150m})\frac{kg}{m^3} \text{ resp. } = 1.752\exp(-\frac{h}{6710m})\frac{\mathrm{kg}}{\mathrm{m}^3} \text{ for } h < (\geq) 9150m \tag{6}$$

It is known that these aerodynamic forces are proportional to $\frac{1}{2}\rho\|v\|^2$. One distinguishes between the force in the direction of the velocity vector a_d, the so-called drag, and the lift perpendicular to the velocity a_t and a_c. Both define the aerodynamic acceleration

$$\vec{a}_A = \frac{1}{2}\rho v^2(-a_d\vec{e}_v + a_t\vec{e}_t + a_c\vec{e}_c) \approx \frac{1}{2}\rho v^2 T_{ENU}^{VTC}\begin{pmatrix}-a_d & a_t & a_c\end{pmatrix}^{\mathrm{T}} \tag{7}$$

Ignoring lift, the (zero-lift) drag component a_d depends on the target mass m, reference area S_{ref} and drag coefficient C_d. Its inverse is well-known as ballistic coefficient [Co94, Ra99]:

$$a_d = \frac{S_{ref}C_d}{m} = \frac{1}{\beta} \tag{8}$$

3 Sequential Monte Carlo Methods

The tracking problem is described by a prediction and measurement equation. The prediction is given by integration of (3) and the usage of (1) and (2):

$$\vec{x}_{t_k} = \int_{t_{k-1}}^{t_k}\vec{f}(t,\vec{x})dt + \vec{q} \quad \text{and} \quad \vec{y}_{t_k} = \vec{h}(\vec{x}_{t_k}) + \vec{w} \tag{9}$$

with $h = T_{ENU}^{RUV}$, measured plot $\vec{y}_{t_k}$ and Gaussian process and measurement noise $\vec{q}$ and $\vec{w}$. To take care of the unknown aerodynamic parameters, one augments the state space by the aerodynamic acceleration, i.e. one substitutes $\vec{x} = \begin{pmatrix}\vec{r} & \vec{v}\end{pmatrix}^{\mathrm{T}} \rightarrow \vec{x} = \begin{pmatrix}\vec{r} & \vec{v} & \vec{a}_A\end{pmatrix}^{\mathrm{T}}$.

3.1 Initialisation

For the initialisation one assumes a uniform distribution of the aerodynamic accelerations and Gaussian position and velocity distribution form the first two plots.

$$\vec{x}_k^i \sim p_0(\vec{x}_0) \tag{10}$$

3.2 Importance Sampling via Numerical Integration

For this step, one has to use the numerical integration techniques to integrate the dynamic equation (3). This is done by the Runge Kutta method given by:

$$\vec{x}_k(t+\Delta t) \approx \vec{x}_{k-1}(t) + \tfrac{\Delta t}{6}(\vec{k}_1 + 2\vec{k}_2 + 2\vec{k}_3 + \vec{k}_4) \tag{11}$$

$$\begin{aligned} &\vec{k}_1 = \vec{f}(t_{k-1}, \vec{x}_{k-1}), && \vec{k}_2 = \vec{f}(t_{k-1} + \tfrac{\Delta t}{2}, \vec{x}_{k-1} + \tfrac{\Delta t}{2}\vec{k}_1), \\ &\vec{k}_3 = \vec{f}(t_{k-1} + \tfrac{\Delta t}{2}, \vec{x}_{k-1} + \tfrac{\Delta t}{2}\vec{k}_2), && \vec{k}_3 = \vec{f}(t_{k-1} + \Delta t, \vec{x}_{k-1} + \Delta t\vec{k}_3) \end{aligned} \tag{12}$$

Afterwards a sampling takes place and weight are calculated:

$$\vec{x}_k^{i\,\prime} \sim p(x_k \mid x_{k-1}^i) \qquad \text{and} \qquad \tilde{w}_k^i = p(z_k \mid x_k^i) w_{k-1}^i \tag{13}$$

3.3 Resampling

One normalises the weights and defines their accumulated weight sums

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{i=1}^{N} \tilde{w}_k^i} \qquad \text{and} \qquad s_i = \sum_{j=1}^{i} w_k^i \tag{14}$$

For a given sequence $(u_j)_{j=1,\ldots,N}$ with $u_1 \sim U\left[0, \frac{1}{N}\right]$ and $u_j = u_1 + \frac{j-1}{N}$ for $j > 1$ a resampling is performed according to:

$$\vec{x}_k^j = \vec{x}_k^{\max\{i|u_j > s_i\}} \tag{15}$$

5 Simulation Results

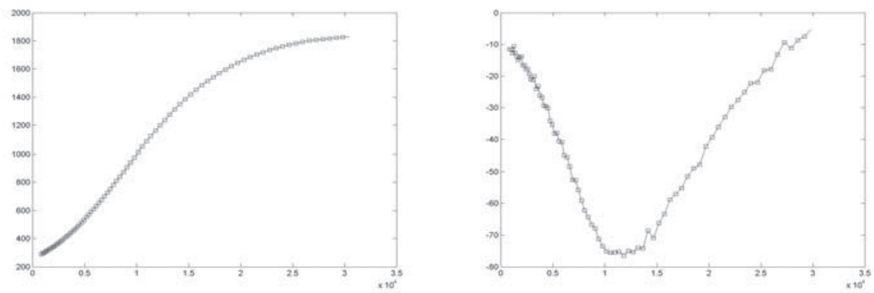

Figure 2: Velocity and acceleration against height of a reentry vehicle.

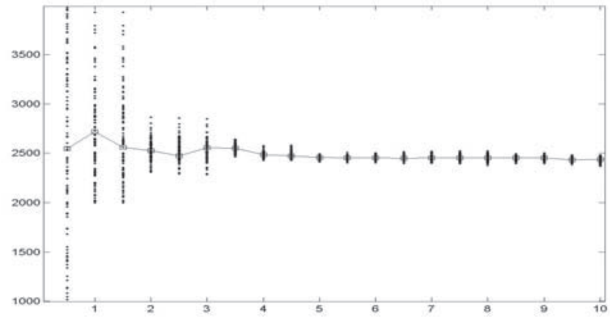

Figure 3: Ballistic coefficient estimation and particles.

The simulation was calculated with (only) 1000 particles for a reentry vehicle with $\beta = 2450 \frac{kg}{m^2}$ including noise. Through the reentry an accelaeration of -7g is calculated. Fig. 2 shows the velocity and acceleration against the height of the object. Fig. 3 gives an impression of the classification capabilities with respect to the ballistic coefficient. Only every 10th particle is shown. For more details on particle filters see [DF01, RA04].

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