Bearings Only Versus Bearings and Extent Tracking for Missile Guidance Systems Utilising Particle Methods

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Abstract: This paper addresses the problem of bearings only and bearings and extent tracking for closed loop missile guidance systems using particle methods compared with derivative free methods. Lack of observability is a typical phenomenon for the bearings only tracking problem. We demonstrate that exploiting angular extent information does help mitigating the lack of observability issues. The simulation results show that the particle filter ourperforms the derivative free algorithms.

1 Motivation

In recent years there has been considerable interest in the application of particle filters and derivative-free filters to target tracking problems, e.g. [ASS03, FRB02]. However, with a few exceptions [Gat09, SEG01], the applications are essentially open-loop; control is largely ignored. Here the application is a closed loop missile homing guidance problem.

For missile homing guidance, an estimator is required in order to calculate states relating to the relative motion of the target with respect to the missile. Traditionally Kalman Filters, or Extended Kalman Filters (EKFs), are used. However, when significant system dynamics or measurement non-linearities exist, and non Gaussian system or measurement noise is present, it is well known that Kalman Filtering has its limitations. Such conditions exist in homing guidance applications. Seeker measurements can have significant non-linearities, and target manoeuvres, can often be best described by non-linear models with non-Gaussian noise. In addition what a missile sees affects what it does, and what it does affects what it sees. Thus any 'parasitic' errors within the system cause missile body-motion to corrupt seeker measurements. In this situation it is not the open loop accuracy of the estimation that is of prime importance, but how errors build up. Closed-loop stability becomes of primary importance. Generally this limits the innovation gains and associated bandwidth of conventional estimators. A further restriction on bandwidth is that noise propagation onto the missile control surfaces must be kept within bounds. However, if the filter model is well matched to the 'real-world' dynamics/measurements good estimation may be obtained using much lower filter gains (or their equivalent). Alternative approaches such as particle filtering or derivative-free techniques are worth exploring.

This work investigates the performance of particle filters (sequential Monte Carlo based methods) within such a closed loop environment, and to what extent the resulting system can be characterised and tuned to be robust to parameter changes. In particular a generic particle filter and the application of so called derivative-free methods in efficient and stable square root form is investigated. Such a technique was developed by MBDA in the late 80's [Vor92]. There now exist a wide variety of similar methods which also aim to improve statistical accuracy. Different approaches to the derivation of these filters are proposed such as use of the unscented transformation leading to the Square Root Unscented Kalman Filter (SRUKF) [WM01] or use of a divided-difference formulations [NPR04]. Here the SRUKF is used.

The problem considered is strap-down sightline-rate estimation for missile homing guidance. The missile carries a seeker and inertial instruments fixed to the missile body (gyros and accelerometers) that in effect allow it to measure the angular bearing of a target. The further case is considered where it is also assumed that onboard signal processing allows the angular extent of some fixed feature on the target to be measured. This is potentially useful extra information particularly if, as here, the problem is restricted to that of homing onto a fixed asset. Without extent information the problem is a bearings only problem. Bearings only tracking has been a problem widely studied in the literature in different contexts [ASS03, FRB02, BC07, CVY05], but mainly for open loop systems.

The remaining part of this paper is organised as follows. Section 2 presents the state equations for the missile guidance system and the measurement model. The proposed approximate solutions to the filtering problem for closed loop guidance are outlined in Section 3. Results are presented in Section 4. Finally, Section 5 concludes the results.

2 System Dynamics and Observation Model

The state vector is chosen as follows: $\boldsymbol{x} = \left(\psi_s - \psi_b, \dot{\psi}_s, 1/R, v_c/R, d/R\right)'$, where ψ_s is the sightline angle, ψ_b is the missile body angle, $\dot{\psi}_s$ is the sightline rate, R is the relative distance of the missile with respect to the target, v_c is the closing speed, d is a fixed distance and $'$ is the transpose operation. Figure 1 a) shows the angles in the engagement geometry. The ratio d/R represents the angular extent of some feature on the target (d is unknown however is 10 metres in the example). Figure 1 b) illustrates this. The system dynamics may be described by the following equations

$$\frac{d(\psi_s - \psi_b)}{dt} = \dot{\psi}_s - \dot{\psi}_b, \tag{1}$$

$$\frac{d\dot{\psi}_s}{dt} = 2\left(\frac{v_c}{R}\right)\dot{\psi}_s - a_m \cos(\psi_s - \psi_b))\left(\frac{1}{R}\right), \tag{2}$$

$$\frac{d\left(\frac{1}{R}\right)}{dt} = \frac{1}{R}\left(\frac{v_c}{R}\right), \tag{3}$$

$$\frac{d\left(\frac{v_c}{R}\right)}{dt} = \left(\frac{v_c}{R}\right)^2 + (a_m \sin[(\psi_s - \psi_b)])\,\frac{1}{R}, \tag{4}$$

$$\frac{d\left(\frac{d}{R}\right)}{dt} = \left(\frac{v_c}{R}\right)\left(\frac{d}{R}\right). \tag{5}$$

A modified polar coordinate system is used which tends to prevent sightline rate estimates needed for guidance being corrupted when range is uncertain or unobservable. Note that

$\left(\frac{v_c}{R}\right) \approx \left(\frac{1}{t_{go}}\right)$ where $t_{go,k}$ is the time to go. A digital approximation to the state equations over a sample time T (here 0.01 sec) is used within the filtering. $\dot{\psi}_b T$ and $a_m T$ are provided by the gyro and accelerometer (here assumed accurate). The seeker measurement

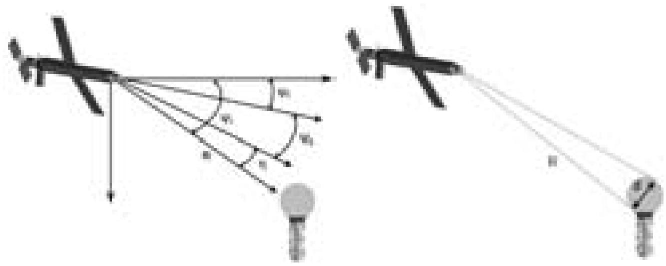

Figure 1: a) Definition of Angles for Sightline Estimation, b) Angular Extent

vector is defined by $\boldsymbol{z}_k \in \mathbb{R}^4$ consisting of: $\boldsymbol{z}_k = (\eta, R, v, d/R)'$, where η is the boresight error. The measurement equations are written in the form:

$$z_{1,k} = \eta + \left(\frac{R}{R_{ref}}\right)^{stype} \nu_{1,k}, \tag{6}$$

$$z_{2,k} = R + \nu_{2,k}, \tag{7}$$

$$z_{3,k} = v_c + \nu_{3,k}, \tag{8}$$

$$z_{4,k} = \left(\frac{d}{R}\right) + \nu_{4,k}. \tag{9}$$

The primary seeker measurement is of boresight error $\eta = (\psi_s - \psi_b) - \psi_g$ where ψ_g is the seeker gimbal angle, assumed to be accurately determined for each measurement time and $\left(\frac{R}{R_{ref}}\right)^{stype} \nu_{1,k}$ is a range dependent boresight error measurement noise term, depending upon whether the seeker type (stype) is passive, semi-active or active assuming values (0,1 and 2) respectively, and ν_k is a vector of noise inputs such that $\nu_k \in \mathbb{R}^4$ and $\nu_k \sim \mathcal{N}(0, \Sigma_{\nu_k})$ that is, normally distributed with zero mean and covariance Σ_{ν_k}.

In this study two types of measurement are considered: *i*) boresight error measurement only (B); *ii*) boresight error and an angular extent measurement d/R (BE). The extent characterises the size of the object and is provided by imagery or radar data.

3 Proposed Solutions

3.1 The Particle Filter

Particle filters [GSS93, AMGC02] have been successfully applied to many nonlinear estimation problems [Gat09, SEG01]. The strength of particle methods, which subsume the sigma point filters, consists in their ability to represent multi-modal posterior densities by a finite set of point masses (particles). The particles are propagated through the true system state and measurement equations rather than approximation of these functions based on linearisation. The posterior probability density function is represented by a weighted statistical sample, from which the distribution's statistics can be approximated. The way in which the point masses are selected and how the weights are ascribed are the principal difference between the algorithms in this class of filter. Hence, the main steps of this approach are prediction and update of the samples, followed by a resampling stage aimed at introducing diversity in the samples.

3.2 The Square Root Unscented Kalman Filter

The Square Root Unscented Kalman Filter (SRUKF) [WM01] is a deterministic Minimum Mean Square Error estimator which propagates the square root of the *a posteriori* covariance rather than the full covariance matrix. QR decomposition and Cholesky updating replace weighted sums in the one step prediction and measurement update equations. The SRUKF has numerical stability as well as computational complexity advantages over non-square root form, with a reduced dynamic range and assured positive definiteness of the covariance matrix. Sigma points filters select the regression points deterministically, and are therefore not Monte Carlo methods.

4 Performance Analysis

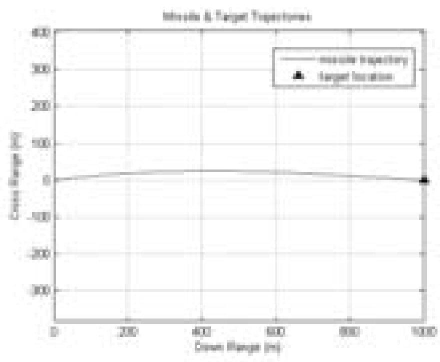

Figure 2: Trajectory of the missile with a static target

The simulated engagement geometry is shown in Figure 2. The seeker type utilised in these simulations is passive with no range dependency in the boresight error. The target is **static**. The filters are initialised with 10% perturbed initial conditions for the range and range rate compared with their true values.

4.1 Results Comparative Analysis of Filter Performance

Figure 3 shows the Root Mean Square Error (RMSE) between the estimated state value and the true values over 200 independent Monte Carlo runs for: a) boresight error and b) sightline rate. Similarly Figure 4 shows RMSE results for: a) range and b) time-to-go, respectively. In respect of boresight error measurement the SRUKF(B) performs worst. The addition of extent information promotes the SRUKF to the top position. However, the PF(B and BE) give similar results. Notice that with bearings only information the PF gives the best results. Similar conclusions apply based on sightline rate estimation.

Results for time-to-go estimation show that the PF(BE) starts to converge towards the

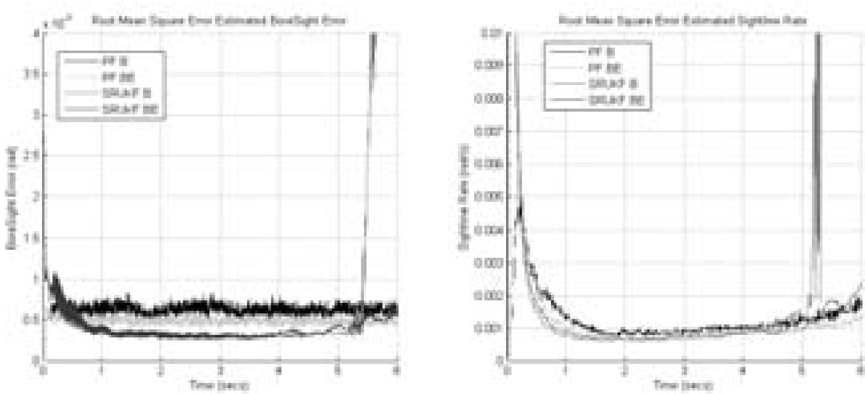

Figure 3: a) RMSE for boresight error, b) RMSE for sightline rate for PF and SRUKF

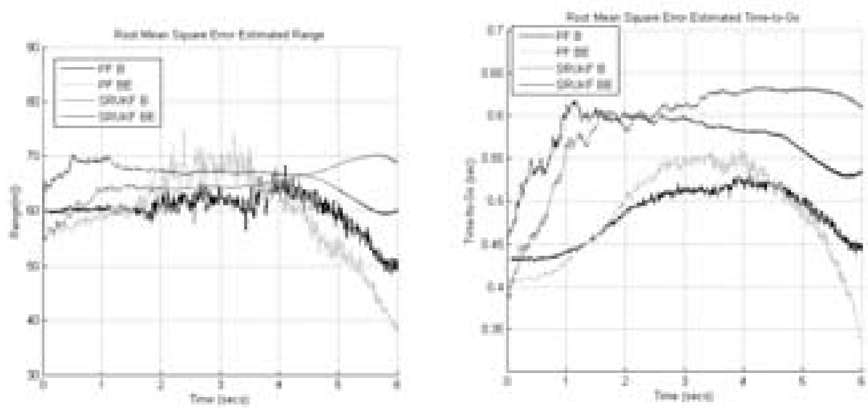

Figure 4: a) RMSE for range, b) RMSE for time to go for PF and SRUKF

end of the engagement. The filters PF(B) and SRUKF(B) do not converge as one might expect because range and closing velocity are not observable. For range and time-to-go estimation the PF(B and BE) clearly outperform the derivative methods.

5 Conclusions

Bearing and extent measurements can help in mitigating the lack of observability phenomenon. The angular extent information enhances by performance of all filters (PFs and UKFs). The experiments show that boresight error and sightline rate estimation are comparable for PF and SRUKF. Particle methods and other derivative free filters provide a computationally efficient approximate solution to the filtering problem.

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