Tracking of Extended Objects and Group Targets using Random Matrices – A Performance Analysis

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Abstract: The task of tracking extended objects or (partly) unresolvable group targets raises new challenges for both data association and track maintenance. Due to limited sensor resolution capabilities, group targets (i. e., a number of closely spaced targets moving in a coordinated fashion) may show a similar detection pattern as extended objects, namely a varying number of detections whose spread is determined by both the statistical sensor errors as well as the physical extension of the group or extended object. Different tracking approaches treating these situations have been proposed where physical extension is represented by a symmetric positive definite random matrix. In this paper, a recently published Bayesian approach is discussed with regard to the estimator's self-assessment of the estimation error for both kinematics and extension.

1 Introduction

In many tracking applications, the objects to be tracked are considered as point sources, i. e., their extension is assumed to be neglectable in comparison with sensor resolution and error. With ever increasing sensor resolution capabilities however, this assumption is no longer valid, e. g., in short-range applications or for maritime surveillance where different scattering centers of the objects under consideration may give rise to several distinct detections varying, from scan to scan, in both number as well as relative origin location. From the associated data – assuming that the related association problem has been solved – we cannot only estimate the kinematic state of the object but also its extension (honoring the spread of the data in comparison with the expected statistical sensor error). But, more than these quantities cannot safely be estimated as well in the (opposite) case where limited sensor resolution causes a fluctuating number of detections for a group of closely spaced targets and thus prevents a successful tracking of (all of) the individual targets.

Several suggestions for dealing with this problem can be found in literature. For an early work, see [DBP90], for an overview of existing work up to 2004, refer to [WD04]. In [Koc06, Koc08], a new and promising suggestion has been introduced by the distinction between kinematical state (a random vector) on the one hand and physical extension (represented by a random matrix) on the other. In order to circumvent some of the problems one may face when applying the Bayesian group tracking approach under circumstances where the underlying assumptions of [Koc06, Koc08] do not hold, a new approach to tracking of extended objects and group targets using random matrices has been proposed in [FF08, FF09]. In the following, we start with a short summary of this approach and subsequently analyze the estimator's self-assessment of the estimation error for both kinematics and extension.

2 Tracking of Extended Objects and Group Targets

The Bayesian approach to tracking extended objects and group targets in [FF08, FF09] adds to the kinematic state of the centroid described by the random vector $\mathbf{x}_k$ the physical extension represented by a symmetric positive definite (SPD) random matrix $\mathbf{X}_k$ thus considering some ellipsoidal shape. It is assumed that in each scan k there are n_k independent position measurements $\mathbf{y}_k^j = \mathbf{H}\mathbf{x}_k + \mathbf{w}_k^j$ where the random vector $\mathbf{x}_k$ denotes the state to be estimated (for us, position and velocity in two or three spatial dimensions) and $\mathbf{y}_k$ the actual measurement (in the following, position only, i. e., $\mathbf{H} = [\mathbf{I}_d, \mathbf{0}_d]$ with $d = 2, 3$). In this paper, we will use the abbreviations $\mathbf{Y}_k := \{\mathbf{y}_k^j\}_{j=1}^{n_k}$ and $\boldsymbol{\mathcal{Y}}_k := \{\mathbf{Y}_\varkappa, n_\varkappa\}_{\varkappa=0}^{k}$ to denote the set of the n_k measurements in a particular scan and for the sequence of what is measured scan by scan, respectively. Now, expected sensor reports are considered as measurements of the centroid scattered over group extension. Having regard to a statistical sensor error of each individual measurement, $\mathbf{w}_k^j$ is assumed to be a zero mean normally distributed random vector with variance $\mathbf{X}_k + \mathbf{R}$. This decisive assumption allows for an estimation of the extension from sensor data based on the corresponding likelihood that reads $p(\mathbf{Y}_k \,|\, n_k, \mathbf{x}_k, \mathbf{X}_k) = \prod_{j=1}^{n_k} \mathcal{N}(\mathbf{y}_k^j; \mathbf{H}\mathbf{x}_k, \mathbf{X}_k + \mathbf{R})$. The relevant probability distributions of this paper are summarized in Figure 1. With the mean measurement $\overline{\mathbf{y}}_k$ and the measurement spread $\overline{\mathbf{Y}}_k$ as defined in Figure 2, it is easily shown that the measurement likelihood can be written as

$$p(\mathbf{Y}_k \,|\, n_k, \mathbf{x}_k, \mathbf{X}_k) \propto \mathcal{N}\big(\overline{\mathbf{y}}_k; \mathbf{H}\mathbf{x}_k, \tfrac{\mathbf{X}_k + \mathbf{R}}{n_k}\big)\, \mathcal{W}(\overline{\mathbf{Y}}_k; n_k - 1, \mathbf{X}_k + \mathbf{R})\,. \tag{1}$$

It appears that, for this likelihood, no conjugate prior can be found that is both independent of $\mathbf{R}$ and analytically traceable. For this reason, [Koc06, Koc08] ignores the sensor error, i. e., setting $\mathbf{R} = \mathbf{0}$, to derive a closed form solution within a Bayesian framework.

$$\mathcal{N}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = |2\pi\boldsymbol{\Sigma}|^{-1/2} \exp\big(-\tfrac{1}{2}[\mathbf{x}-\boldsymbol{\mu}]^T \boldsymbol{\Sigma}^{-1}[\mathbf{x}-\boldsymbol{\mu}]\big) \qquad E[\mathbf{x}] = \boldsymbol{\mu} \qquad \mathrm{Var}[\mathbf{x}] = \boldsymbol{\Sigma}$$

$$\mathcal{W}_d(\mathbf{X}; m, \mathbf{C}) = \big(2^{md/2}\Gamma_d(\tfrac{m}{2})\,|\mathbf{C}|^{m/2}\big)^{-1} |\mathbf{X}|^{(m-d-1)/2}\,\mathrm{etr}\big(-\tfrac{1}{2}\mathbf{X}\mathbf{C}^{-1}\big)$$
$$E[\mathbf{X}] = m\mathbf{C} \qquad \mathrm{Var}[\mathbf{X}] = m\mathbf{C}^2 + m\,\mathrm{tr}(\mathbf{C})\mathbf{C} \qquad (m \geq d)$$

$$\mathcal{IW}_d(\mathbf{X}; \nu, \alpha\mathbf{M}) = \big(2^{\nu d/2}\Gamma_d(\tfrac{\nu}{2})\,|\mathbf{X}|^{(\nu+d+1)/2}\big)^{-1} |\alpha\mathbf{M}|^{\nu/2}\,\mathrm{etr}\big(-\tfrac{\alpha}{2}\mathbf{M}\mathbf{X}^{-1}\big)$$
$$(\nu = \alpha + d + 1) \qquad E[\mathbf{X}] = \mathbf{M} \qquad \mathrm{Var}[\mathbf{X}] = \frac{\alpha\,\mathrm{tr}(\mathbf{M})\mathbf{M} + (\alpha+2)\mathbf{M}^2}{(\alpha+1)(\alpha-2)} \qquad (\alpha > 2)$$

Figure 1: Distributions of *normal*, *Wishart* and *inverse Wishart density*, see [GN99].
$\mathbf{X}$ and $\mathbf{M}$ are d-dimensional SPD (random) matrices, $\mathrm{etr}(\cdot)$ is an abbreviation for $\exp(\mathrm{tr}(\cdot))$ and Γ_d is the multivariate gamma function.

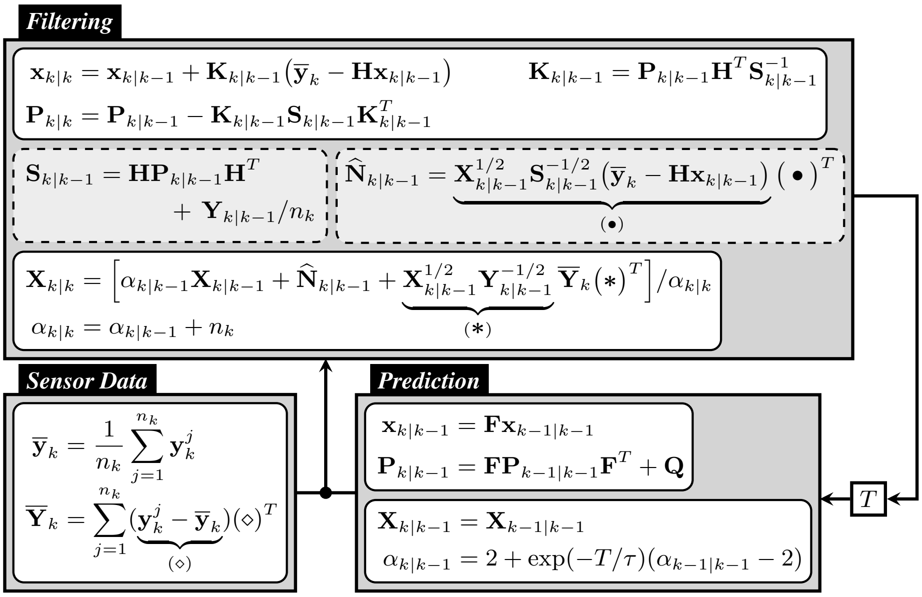

Figure 2: Bayesian Formalism.
$\mathbf{Y}_{k|k-1} = \mathbf{X}_{k|k-1} + \mathbf{R}$ denotes the predicted variance of a single measurement. Some square roots (e. g., via Cholesky decomposition) of the matrices $\mathbf{X}_{k|k-1}$, $\mathbf{S}_{k|k-1}$, and $\mathbf{Y}_{k|k-1}$ are required.

In [FF08], several implications of the neglect of any (unavoidable) statistical sensor error have been brought to attention. Among other things, the algorithm effectively estimates extension *plus* sensor error, which consequently, if sensor errors become significant, leads to a more than proportionally increased centroid estimation error because there is an almost fixed (up to a scalar constant) coupling between the estimated extension and the mean squared position estimation error.

In view of these observations, a new approach has been sought that allows reliable tracking of extended objects and group targets in cases where sensor errors cannot be ignored when compared with object or group extension. Using some careful approximations, the proposed alternative approach in [FF08, FF09] honors the fact that both sensor error and extension contribute to the measurement spread. In detail, the proposal may be interpreted as approximating the marginal densities of the joint object state $(\mathbf{x}_k, \mathbf{X}_k)$ according to $p(\mathbf{x}_k \,|\, \boldsymbol{\mathcal{Y}}_k) \approx \mathcal{N}(\mathbf{x}_k; \mathbf{x}_{k|k}, \mathbf{P}_{k|k})$ and $p(\mathbf{X}_k \,|\, \boldsymbol{\mathcal{Y}}_k) \approx \mathcal{IW}(\mathbf{X}_k; \alpha_{k|k} + d + 1, \alpha_{k|k}\mathbf{X}_{k|k})$, which finally leads to the Bayesian formalism summarized in Figure 2. The dashed boxes highlight the matrices $\mathbf{S}_{k|k-1}$ and $\widehat{\mathbf{N}}_{k|k-1}$, which provide for the interdependency between kinematics and extension estimation in the filtering step. Simulation results have shown that this approach can compensate significant sensor errors to a large extent and thus, although compensation is not complete, may be able to, e. g., detect orientation changes of formations in cases where the original approach of [Koc06, Koc08] might fail to do so.

3 Performance Analysis

When using estimators, we usually are not only interested in the estimate itself but also in the corresponding estimator's self-assessment of the estimation error. In our case, the stated uncertainty corresponds to the estimation error covariance and the mean square error (MSE) respectively. For the kinematics estimate $\mathbf{x}_{k|k}$, this estimation error covariance is given by $\mathrm{Var}[\mathbf{x}_k \,|\, \boldsymbol{\mathcal{Y}}_k] = \mathbf{P}_{k|k}$ and therefore the corresponding MSE by $\mathrm{tr}\,\mathbf{P}_{k|k}$. In order to judge the credibility of the kinematics estimate, we exploit the average normalized estimation error squared, i. e.

$$\mathrm{ANEES}_{\mathbf{x}} = \frac{1}{\dim(\mathbf{x}_k) \cdot M} \sum_{\mu=1}^{M} \Big[(\mathbf{x}_{k|k} - \mathbf{x}_k)^T \mathbf{P}_{k|k}^{-1} (\mathbf{x}_{k|k} - \mathbf{x}_k) \Big]_{\mu}, \tag{2}$$

where values larger than 1 indicate that the filter is overly confident about its estimation quality. The subscript μ indicates tracking results concerning the μth run of a Monte Carlo simulation totaling M runs. Transferring this credibility measure to the extension estimate $\mathbf{X}_{k|k}$ requires the MSE $e_{k|k}$, which is computed by summing up the mean square errors obtained for each element of $\mathbf{X}_{k|k}$ with respect to the true extension $\mathbf{X}_k$, see [FF09], and can be written as $e_{k|k} = \mathrm{tr}\,\mathrm{Var}[\mathbf{X}_k \,|\, \boldsymbol{\mathcal{Y}}_k]$. Following the idea of the ANEES to judge the credibility of the extension estimate, we define

$$\mathrm{ANEES}_{\mathbf{X}} = \frac{1}{M} \sum_{\mu=1}^{M} \left[\frac{\mathrm{tr}\left[(\mathbf{X}_{k|k} - \mathbf{X}_k)^2 \right]}{e_{k|k}} \right]_{\mu}, \tag{3}$$

where also values larger than 1 indicate that the filter is overly confident about its estimation quality. In addition to the known relative ($\mathrm{ANEES}_{\mathbf{x}}$, $\mathrm{ANEES}_{\mathbf{X}}$) and absolute errors (target location/speed error), we define

$$\mathrm{RMSE}_{\mathbf{X}} = \sqrt{\frac{1}{M} \sum_{\mu=1}^{M} \Big[\mathrm{tr}\left[(\mathbf{X}_{k|k} - \mathbf{X}_k)^2 \right] \Big]_{\mu}} \tag{4}$$

for the extension part. In order to compute $\mathrm{ANEES}_{\mathbf{X}}$ and $\mathrm{RMSE}_{\mathbf{X}}$, we need to know the true ellipsoid $\mathbf{X}_k$. For this reason, we have considered an ellipse with diameters $340\,\mathrm{m}$ and $80\,\mathrm{m}$ as an extended object in the (x,y)-plane. Such an extended target corresponds approximately to an aircraft carrier of the Nimitz-class[1]. The basis for our simulation was the target trajectory in the upper part of Figure 3, where the speed was assumed to be constant at $27\,\mathrm{kn}$ ($\approx 50\,\mathrm{km/h}$). We have chosen two different models to generate measurements. The first corresponded to our assumed measurement likelihood $\mathbf{y}_k^j \sim \mathcal{N}(\mathbf{H}\mathbf{x}_k, \mathbf{X}_k + \mathbf{R})$ and is hereinafter referred to as $\mathcal{N}(\mathbf{X}_k + \mathbf{R})$. The second model $\mathcal{U}(\mathbf{X}_k) + \mathcal{N}(\mathbf{R})$ assumed that measurements were uniformly distributed over the extension $\mathbf{X}_k$ and were additionally afflicted with a zero mean normally distributed sensor error with variance $\mathbf{R}$. Both models generated measurements, where the number of measurements n_k for each scan k was Poisson-distributed with mean 5. Furthermore, the (fictitious) observing sensor with scan time $T = 10\,\mathrm{s}$ delivered uncorrelated noisy x- and y-measurements with standard deviations $\sigma_x = 100\,\mathrm{m}$ and $\sigma_y = 20\,\mathrm{m}$.

[1] `http://en.wikipedia.org/wiki/Nimitz_class_aircraft_carrier`. Retrieved on 2009-04-24.

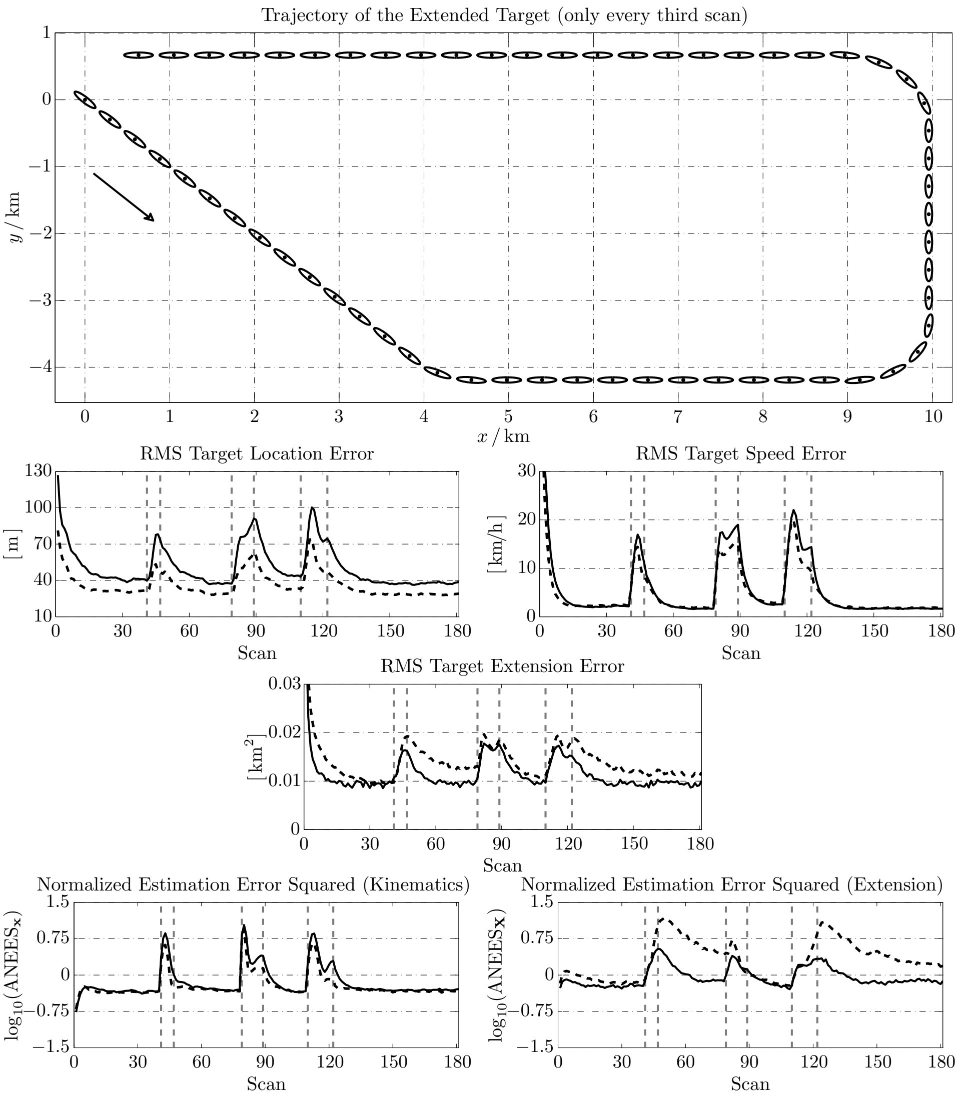

Figure 3: Trajectory of the extended object (top) and simulation results (middle and bottom). The five figures in the bottom part summarize the results of $M = 900$ Monte Carlo runs, where the solid line indicates the measurement generating model $\mathcal{N}(\mathbf{X}_k + \mathbf{R})$ and the dashed one the model $\mathcal{U}(\mathbf{X}_k) + \mathcal{N}(\mathbf{R})$. The dashed vertical lines mark each start and end of the three maneuvers.

The second model $\mathcal{U}(\mathbf{X}_k) + \mathcal{N}(\mathbf{R})$ seems to be more realistic in view of extended objects. But, for an appropriate extension estimate in this case, we have to consider a scaling factor z, which can be directly integrated into the measurement likelihood by replacing $\mathbf{X}_k$ with $\mathbf{X}_k/z$ in the right side of eq. (1). As a matter of course, this replacement affects the tracking filter equations. For example, the predicted variance of a single measurement in Figure 2 is modified according to $\mathbf{Y}_{k|k-1} = \mathbf{X}_{k|k-1}/z + \mathbf{R}$.

The simulation results are summarized in the bottom part of Figure 3. First of all, it attracts attention that the filter is too optimistic about its estimation quality during maneuver phases despite the fact that we have used the refined interacting multiple model (IMM) approach of [FF09]. Similar peaks can be understandably discovered in the absolute errors (target location/speed error, target extension error), where the measurement generating model $\mathcal{N}(\mathbf{X}_k + \mathbf{R})$ causes smaller absolute errors than $\mathcal{U}(\mathbf{X}_k) + \mathcal{N}(\mathbf{R})$ for the kinematics part. This trend is quite contrary to the extension part. Furthermore, in the case of $\mathrm{ANEES}_{\mathbf{X}}$, the measurement generating model $\mathcal{U}(\mathbf{X}_k) + \mathcal{N}(\mathbf{R})$ involves a little too optimistic self-assessment of the estimation error. This result is quite remarkable because, in our opinion, the estimation error variance of the extension part will play a decisive role in future developments of data association strategies. This raises the interesting question of what an acceptable estimation error is, when there is no real ellipsoidal extension $\mathbf{X}_k$ (e. g. aircraft formations, convoys, ...).

4 Conclusion

A recently published approach to tracking of extended objects and group targets has been discussed with regard to the estimator's self-assessment of the estimation error for both kinematics and extension. Aside from more intensive performance studies with real sensor data concerning a limited sensor resolution, the most challenging task of future work will be the derivation of sophisticated data association techniques.

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