Stochastic Evaluation of Geographical Forwarding in Vehicular Ad Hoc Networks

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Abstract: Vehicular ad hoc networks which support active safety applications have to address data packet recipients by taking the position of the vehicles into account. This leads to a two step forwarding procedure. Firstly, in line forwarding mode data packets are sent to a target area and in a second step the data packets are distributed inside the target area (area forwarding mode). This contribution introduces methods of stochastic analysis to estimate the performance of these two forwarding phases.

1 Introduction

Driving means constantly changing the location. Since information on the current location is required for safety and comfort reasons all the time, a continuous demand for location-dependent information exists. Location-dependent information with respect to inter-vehicle communications includes sensor data from other cars such as data on braking events sent from a preceding car (active safety applications), data on the traffic flow on a specific route, and information on sites located along the route. The specification of such an inter-vehicle communication system was the objective of the project "FleetNet – Internet on the Road", which was partly funded by the German Ministry of Education and Research (BMB+F) [FEL01], [FN01].

Since data communication in vehicular ad hoc networks is limited by the communication range of the radio system, multihop communication has to be applied. For that reason routing and forwarding protocols have to be specified and employed in each vehicle. By means of simulation it was found that by taking the position of the vehicles into account, multihop data transmissions in highly dynamic vehicular networks can be supported efficiently [FM02]. Active safety applications generally address data packet recipients by their physical location. For this reason in FleetNet a geographical addressing scheme was defined and the forwarding of data packets to the recipients was consequently split into two phases [FW04]. In the line forwarding phase data packets are forwarded to the so-called target area, which is the area in which the recipients are located. In a second step (area forwarding) the data packets are distributed inside the target area, either to all vehicles inside this area (geo-broadcast), or to one recipient addressed by an additional unique identifier (geo-unicast), or to an arbitrary vehicle (geo-anycast).

The routing scheme developed in FleetNet is based on the Greedy forwarding as introduced in [KK00], [MW01]. However, in contrast to [KK00] a new location lookup algorithm was defined which takes into account the movement pattern of vehicles.

Moreover, the perimeter mode is not applied, since simple packet caching strategies lead to better results [FM02], [ME04]. For example, to send a data packet to a recipient which position is only roughly known by the sender (geo-unicast), the sender will define a target area, in which the recipient is supposed to be. Then the data packet is forwarded to this target area. If the first receiving node inside the target area doesn't know the recipient's position, it will start the location lookup service. Compared to a location lookup around the sender's position, the location lookup is limited to a much smaller area, which increases network efficiency. Additionally, the probability that a node close to the recipient already knows the recipient's position is very high. In this case the location lookup algorithm will not be started and the network performance is increased again. As soon as the first node inside the target area knows the recipient's position, the data packet is sent to this position by running the Greedy forwarding algorithm.

In this paper, both the performance of the line forwarding and the area forwarding phases are computed by means of stochastic analysis (Chapter 2). Chapter 3 shows results of representative scenarios.

2 Stochastic Analysis of Line and Area Forwarding

2.1 Model, Parameter Definitions, and Assumptions

In this chapter methods are presented to estimate the efficiency of the FleetNet network layer by computing the spatial progress of the data packets at a one hop transmission. To describe the model mathematically we assume in line and area forwarding mode a constant density and a uniform distribution of vehicles (ρ_{LFW} *[veh/m]*, ρ_{AFW} *[veh/m²]*) and furthermore a fixed maximum transmission range (r).

In the line forwarding mode the random value X describes the distance between pairs of neighboring vehicles. It is assumed that the vehicles are located along a one-dimensional line, e.g. a piece of a motorway. The line is starting at the senders location at $x = 0$. The probability density function for a node to be located at position x is denoted as $a(x)$, the distribution function as $A(x)$. $a(i,x)$ denotes the density probability of the i-th car to be at position x.

In the area forwarding mode the distance between the next forwarding node and the destination is the most important parameter according to the greedy forwarding protocol (Figure 1). The GPRS area is the area in which possible forwarding nodes are located. According to the Greedy forwarding approach, this is the area which is defined by the communication range of the sender (r) and the area in which the vehicles are closer to the destination's position than the sender itself. Possible forwarders are the black nodes in Figure 1 inside the gray colored GPSR-area. The random variable Y describes the distance of different possible forwarders in the GPRS area to the sender. Please note that this definition is different to the definition of X in line forwarding.

2.2 Derivation of Formulas

Starting from the distance between two vehicles (random variable *X*) in **line forwarding mode**, we compute first the density function of the distance between sender and the *i*-th vehicle *a(i,x)* which is a sum of identical distributed random variables. For this *a(x)* has to be convoluted with itself i-times. In the next step the probability function *b(i,x)* is derived, which takes into account that additionally no other car is located between *x* and *r* (distance between i-th and *(i+1)*-th vehicle is larger than $r-x$).

$$b(i,x)=a(i,x)\cdot P_1(X_{i+1}>r-x)\ ,\ P_1(X_{i+1}>r-x)=1-A(r-x)\ ,\ A(x)=\int_0^x a(\xi)d\xi\ ,\ x\in[0,r]$$

b(i,x) describes the probability density function that the vehicle closest to position *r* is at position *x* and *i* vehicles are in communication range of the sender.
The spatial progress of the packet by one transmission is the expected value of $b(i,x)$:

$$E_{LFW}(i)=\int_0^r x\cdot\frac{b(i,x)}{B(i)}dx \text{ with } B(i)=\int_0^r b(i,x)dx=\int_0^r a(i,x)\cdot[1-A(r-x)]dx$$

$$E_{LFW}=\sum_{i=1}^{\infty}E_{LFW}(i)\cdot B(i)=\sum_{i=1}^{\infty}\int_0^r x\cdot b(i,x)dx$$

In the next step the **area forwarding mode** will be investigated. As described in [Wa04] the Poisson density function $n(i)$ describes the probability for a certain number of vehicles in the area A_0 with $n(i)=\frac{\mu^i}{i!}\cdot e^{-\mu}$ and $\mu=\rho_{AFW}\cdot A_0$.

The size of the GPSR area is described by:

$$A_{GPSR}=r^2\cdot\arccos\left(\frac{r}{2\cdot d}\right)+2\cdot d^2\cdot\arcsin\left(\frac{r}{2\cdot d}\right)-r\cdot d\cdot\sin\left(\arccos\left(\frac{r}{2\cdot d}\right)\right) \quad \text{([Wa04])}$$

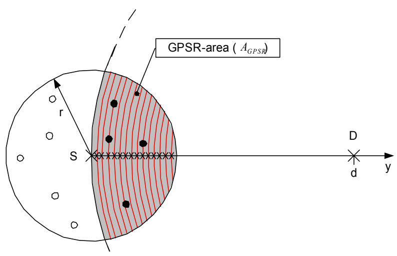

Figure 1. Area Forwarding

Parameter d describes the distance between the sender *S* and the destination *D*. Vehicles inside the GPSR area which have the same distance to the destination *D* are located on segments of circles with central point *D*. Projecting the length of these circle segments to the point of intersection (crosses in Figure 1) with the line from *S* to *D*, we achieve a continuous function *f(y)*, which is proportional to the density function of the probability of a forwarder with distance (*d-y*) to *D*.

The length of the circle segments depend on *y* and are calculated as follows:

$$l(y)=2\cdot(d-y)\cdot\arccos\left(\frac{y^2-2\cdot d\cdot y+2\cdot d^2-r^2}{2\cdot d\cdot(d-y)}\right),\ y\in[0,r]$$

To achieve the density function $f(y)$, $l(y)$ between *0* and *r* at has to be related to the expected sum of all vehicles inside the GPSR area. Then we get:

$$f(y)=\frac{2}{C}\cdot(d-y)\cdot\arccos\left(\frac{y^2-2\cdot d\cdot y+2\cdot d^2-r^2}{2\cdot d\cdot(d-y)}\right) \text{ with } C=\int_0^r l(y)dy$$

The calculated density function is valid for every single vehicle inside the GPSR area, because the vehicles are uniform distributed over a large area and therefore their positions are independent from each other. The position of the vehicle with the largest spatial progress of the data transmission is described by the maximum of the random values *Y* for all vehicles inside the GPRS area. The density function *g(i,y)* of the spatial progress and *i* vehicles inside the GPRS area is:

$$g(i,y) = i \cdot \left[F(y)\right]^{i-1} \cdot f(y) \text{ with } F(y) = \int_0^y f(\psi)d\psi$$

$g(i,y)$ is the probability density function for the location of the vehicle with the largest progress for the data packet towards D and i vehicles within the GPSR area. The mean progress for the data packet at a one hop transmission is given by the expected value $E_{AFW}(i)$ of $g(i,y)$. By resolving that *i* vehicles are inside the GPSR-area one get:

$$E_{AFW} = \sum_{i=1}^{\infty} E_{AFW}(i) \cdot n(i) \text{ with } E_{AFW}(i) = \int_0^r y \cdot g(i,y)dy$$

This total expected value is still dependent from the distance between sender and destination (*d*). But as shown in [Wa04] an estimation of the expected value for $d \to \infty$ can be made. Investigations on the error occurring for $d \to \infty$ are also presented in [Wa04].

3 Results

In the following example we assume, that the distance between two vehicles in the line forwarding mode is negative exponential distributed. So $a(x) = \lambda \cdot e^{-\lambda \cdot x}$ and $A(x) = 1 - e^{-\lambda \cdot x}$ with $\lambda = \rho_{LFW}$. For the area forwarding a vehicle distribution as described in the previous chapter is computed. The communication range is set to r = 500 m. The results for the physical progress for a data packet in line forwarding mode using the calculations described above is:

$$E_{LFW} = \frac{1 - (1 - \lambda \cdot r) \cdot e^{\lambda \cdot r}}{\lambda} \cdot e^{-\lambda \cdot r}$$

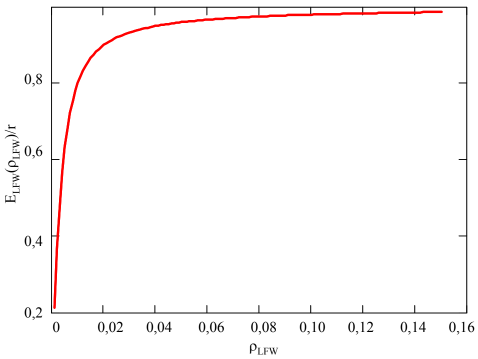

Figure 2. E_{LFW}

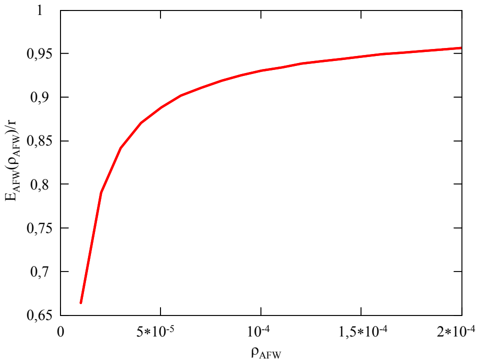

Figure 3. E_{AFW}

The expected values for the progress of a on hop data packet transmission depend on the vehicle density ρ_{LFW} in the Line-Forward-Mode and from the density ρ_{AFW} in the Area-Forward-Mode. This results dependent on ρ_{LFW} and ρ_{AFW} are shown in Figure 2 and Figure 3. In the line forwarding mode the expected value of the progress for the data packet is above 90% of the communication range, if the density is higher than 20 vehicles per 1000 m. This density is a realistic figure for a motorway with moderate traffic. But at lower densities, the expected value decreases strongly. This is due to the decreasing probability that at least one vehicle is within the communication range r.
As mentioned, the investigations in the Area-Forwarding-Mode are done with the expected value for the limitation $d \to \infty$. In Figure 3 the curve progression of the expected value is shown. Especially when the vehicle density is low, the probability that an appropriate forwarder is found decreases, which leads to the low physical progress values. To overcome this situation the data packets can be cached in line forwarding mode and geo-unicast transmissions. [ME04] shows that simple caching leads to good packet delivery rates in mobile networks, in case of low vehicle densities. Whereas at higher car densities the average spatial progress is close to the communication range which is shown in this paper.

4 Conclusion

Geographical addressing is needed in vehicular ad hoc networks, since most cooperative driver assistance and active safety applications address their recipients by their position. In this work the efficiency of line and area forwarding is computed by means of stochastic analysis. It is shown that already at moderate traffic density the progress of one data transmission is close to the maximum transmission range.

5 References

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