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Mathematical structures for modeling semantics, uncertainty, and cognitive processes

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Abstract: This talk investigates mathematical structures which are useful for designing cognitive machines with special emphasis on their mathematical properties.

Semantics. Starting point is modeling *semantics* by *order-deterministic pomsets* as defined in [Re96]. It is clear that any syntax tree and any feature structure can be represented by a pomset, hence this model is appropriate for 'literal semantics' as considered in [CEEJKL13, Kapitel 3]. On the other hand, in [WHKLR12] it is suggested that more basic 'pragmatic semantics' as considered in [Sk57, p. 14] and [WL13] are describable by feature-value-relations, and hence also by pomsets.

The *prefix relation* is a partial order on the class of pomsets, or on any set of pomsets over a fixed common alphabet. When we restrict attention to order-deterministic pomsets, then, as is shown in [Re96], the prefix relation has infima and finite suprema. If we have two feature structures which admit *unification* [CEEJKL13, Abschnitt 2.3.3], we obtain this unification by regarding the feature structures as order-deterministic pomsets and taking their supremum.

Uncertainty. For modeling *uncertainty*, consider valuation maps $\Phi : \mathbb{U} \to \mathbb{X}$ defined on a given (finite) set $\mathbb{U}$ of *semantic units*, and associating a *valuation* $\Phi(u) \in \mathbb{X}$ de each semantic unit $u \in \mathbb{U}$ (cf. [WL14]). The valuation $\Phi(u)$ represents uncertainty: when $\mathbb{X} = [0,1]$, an element $x \in \mathbb{X}$ could be interpreted as a probability, or as a fuzzy value. In the present abstract setting, the set $\mathbb{X}$ is not fixed, and other interpretations are also conceivable.

Decision. Now consider also *partial valuation maps* $\Phi : \mathbb{V} \to \mathbb{X}$, where $\mathbb{V} \subseteq \mathbb{U}$ is subset. Denote the domain of definition of a (partial) valuation map by $\mathbb{D}(\Phi)$. Then *decision* is an operation restricting a given (partial) valuation map Φ to some subset

$$\mathbb{D}(\nabla\Phi) \subseteq \mathbb{D}(\Phi) \subseteq \mathbb{U},$$

leading to a partial valuation $\nabla\Phi := \Phi|_{\mathbb{D}(\nabla\Phi)}$ which only valuates the semantic units corresponding to the decision. The elements of $\mathbb{D}(\nabla\Phi)$ are called *winners* of the decision. A decision operation is called *consistent with valuation* [WL14], if there is a binary relation $\rhd$ on $\mathbb{X}$ such that

$$u \in \mathbb{D}(\nabla\Phi) \quad \Leftrightarrow \quad u \in \mathbb{D}(\Phi) \wedge \forall v \in \mathbb{D}(\Phi) : \Phi(u) \rhd \Phi(v). \tag{1}$$

It is proved in [WL14] that, in this situation, properties of ∇ correspond to properties of $\rhd$ as follows:

Decision ∇	**Binary relation $\rhd$**
– non-vanishing	– finite subset-topped
– effective	– antisymmetric
– subset-stable	– transitive

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This implies that a decision operation ∇ which is consistent with valuation is non-vanishing, effective, and subset-stable, if and only if it is based on a total order $\rhd$ on $\mathbb{X}$ via (1). On the other hand, there is no apparent connection between $\rhd$ and the prefix relation on $\mathbb{U}$.

Join. The second cognitive operation considered here is 'join' of semantic units, and what this means for valuation. To fix notation, let $\Phi_1 : \mathbb{U}_1 \to \mathbb{X}$ and $\Phi_2 : \mathbb{U}_2 \to \mathbb{X}$ be two given valuation maps. For semantic units $u_1 \in \mathbb{U}_1$ and $u_2 \in \mathbb{U}_2$, given as order-deterministic pomsets, denote by $u_1 \sqcup u_2$ their *join* [Re96], or, equivalently, their supremum w.r.t. prefix relation. It may happen that some of the joins $u_1 \sqcup u_2$, when u_1 runs over $\mathbb{U}_1$ and u_2 runs over $\mathbb{U}_2$, do not refer to real situations and therefor can be discarded. The remaining 'meaningful' joins are collected in a subset $\mathbb{V} \subseteq \mathbb{U}_1 \sqcup \mathbb{U}_2$. Moreover, as $\sqcup$ is idempotent, it may happen that $\varnothing \neq \mathbb{U}_1 \cap \mathbb{U}_2 \subseteq \mathbb{V}$. Now a valuation map $\Phi : \mathbb{V} \cup \mathbb{U}_1 \cup \mathbb{U}_2 \to \mathbb{X}$ which extends both Φ_1 and Φ_2 is called *consistent with join*, if there is a binary operation $\vee$ on $\mathbb{X}$ such that

$$\forall u_1 \in \mathbb{U}_1 \text{ and } u_2 \in \mathbb{U}_2 : \quad u_1 \sqcup u_2 \in \mathbb{V} \;\Rightarrow\; \Phi(u_1 \sqcup u_2) = \Phi(u_1) \vee \Phi(u_2).$$

If we assume that $\Phi(u_1 \sqcup u_2) = \Phi(u_1) \vee \Phi(u_2)$ is valid for arbitrary order-determinsitic pomsets u_1, u_2, this would imply that $\vee$ is taking the supremum w.r.t. some partial order $\leqslant$ on $\mathbb{X}$, and that Φ respects the prefix relation in the sense that $\Phi(v) \leqslant \Phi(u)$ whenever v is a prefix of u. Note that at this stage, there is no apparent connection to the total order $\rhd$ considered in (1).

Conclusion. Suppose that we are to design a cognitive system, and we decided to use for modeling semantic units elements from the set $\mathbf{DPOM}(\mathbf{E})$ of order-deterministic pomsets over a given alphabet $\mathbf{E}$. If we wish to model uncertainty by a valuation map $\Phi : \mathbf{DPOM}(\mathbf{E}) \to \mathbb{X}$ which behaves well w.r.t. rational cognitive operations, then it suffices to ensure that Φ maps the prefix relation to a total order on $\mathbb{X}$, and the join $\sqcup$ to taking the maximum w.r.t. that total order.

Keywords: Pomset, order-deterministic pomset, prefix relation, distributive lattice, unification, feature structure, uncertainty, semantic unit, valuation map, decision operation, binary relation, join operation, binary operation.

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