An Efficient Transport Capacity Estimation Method for Wireless Multihop Network Topologies

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Abstract: In this paper, we present a method for efficiently evaluating the potential transport capacity of a given wireless multihop network topology.

1 Introduction

Transmission power control in wireless multihop networks is a popular research topic. An interesting motivation is the capacity gain that can be achieved by additional spatial re-use in single-channel networks (where all stations share a single channel), though it is often rather motivated by the need for energy efficiency.

Because simulation results are unable to give deeper insight into the capacity potential of network topology without taking the negative influence of specific MAC, routing or transport protocols into account, we present an efficient method to estimate the potential throughput of a given set of unicast streams in a given topology, assuming a "perfect" protocol stack. Instead of directly simulating packet transmissions, it is based on graph colouring to obtain independent sets of packet transmissions that can take place simultaneously. The resulting throughput values for the individual traffic streams are achievable and approximately conform to the well-defined max-min fairness criterion. We verify our method by comparing its results to those of a high-level network simulator.

The rest of this work is structured as follows: In section 2, we present related work on the capacity of wireless multihop networks. In section 3, we introduce the network model that forms the basis of the capacity estimation. In section 4, our capacity estimation method is described. In section 5, we present some results and compare them to simulation results. Section 6 concludes our paper and outlines perspectives for future research.

2 Related Work

The difficulty of calculating the possible throughput of streams in a wireless multihop topology is due to the fact that the rate feasibility problem in wireless multihop networks is NP-complete [Ar84].

Several purely analytical approaches to evaluate the transport capacity of network topologies with certain characteristics have been published, e.g. [TK84], [GK00]. In this context, general and simplifying assumptions about the given topology must be made. Therefore, these publications give interesting asymptotical results as the number of stations approaches infinity, but these ideas cannot be applied to specific topologies determined by arbitrary power control strategies.

A second category of publications uses network simulators to determine the throughput or the delivery ratio of data traffic. The prevalent network simulators model a specific protocol stack in a very detailed manner, so the results reflect how well certain MAC, routing and transport protocols are able to deal with a certain topology rather than expressing the potential capacity of the topology itself.

To our knowledge, the first approach to estimate the transport capacity of a specific, given topology was just recently published [JPPQ03]. The authors assume that the topology and the traffic pattern (i.e. source and destination pairs) are given. The capacity is estimated as the cumulative throughput of maximal flows, subject to interference constraints represented in a so-called conflict graph. This requires a very high computational effort, which is also reflected by the fact that only results for networks with at most 50 stations are presented. Therefore, even with special software and modern hardware, it is questionable whether this approach is feasible for networks with hundreds or thousands of stations. A main difference to our approach is that routes need not be specified; rather, all possible routes are used. However, we have shown in [GdWFM04] that using specific routes is not a restriction if there is a sufficiently high number of streams.

In [GdWFM04], we have estimated the transport capacity of wireless multihop topologies using high-level simulation based on nearly the same network model we describe in section 3. In that work, we introduce results for network topologies created according to different power control strategies with varying parameter settings. We refer to those results in section 5 to validate the calculations based on our new approach.

There are also several publications dealing with the possible MAC layer throughput (e.g. [Ra97]). This however disregards important requirements of multi-hop streams, e.g. the connectedness between source and destination stations or the severity of bottlenecks. Often, these approaches are based on graph colouring used to calculate independent sets of transmissions that can take place simultaneously. Our approach takes up this idea to calculate feasible data rates for multihop streams.

3 Network Model

Our approach is based on the idea of finding a good transmission schedule assuming an ideal, slotted MAC layer: Packet transmissions take one slot time, and within each slot, simultaneous transmissions take place only if they do not interfere with each other. Interference can be modelled in several ways, as long as one requirement is fulfilled: Whether two transmissions can take place simultaneously may only depend on themselves and not on any other transmissions. Consequently, it is possible to model interference according to the “protocol model” which was used in the calculations presented in section 5, but not according to the “physical model” (both presented in [GK00]).

In our model, all sources are saturated. A bottleneck-aware transport protocol is assumed, meaning that the data rate of a stream is complied with at every intermediate hop; in other words, there are no packet losses due to buffer overflows.

Our algorithm requires one route to be explicitly chosen for each stream. Of course, the choice of specific routes can have a strong impact on the results, but we have shown in [GdWFM04] that with a sufficiently high number of streams, using shortest paths according to a certain metric results in a consistently high throughput that cannot be significantly raised by using additional routes.

Finally, fairness aspects should be taken into account when measuring the transport capacity of a network, because the highest overall throughput is generally achieved under highly unfair and therefore undesired conditions. For this reason, we approximate max-min fairness between different streams.

4 The Two-Phase Colouring Algorithm

Calculating a max-min fair resource allocation for given flows in a single-channel wireless domain is not straightforward. In principle, the well-known water filling technique can be used, as global bottlenecks can be identified by examining link neighbourhoods. However, our previous research has shown that a satisfactory extension of water filling by a scheduling heuristic is hardly possible. Therefore, our approach is designed differently.

Let f be a flow and L_f the set of links used by f. Then every tuple (f,l), $l \in L_f$ is called a *flow link*. An *interference graph* is an undirected graph where the vertex set consists of all flow links, and there is an edge between two vertices if and only if the corresponding transmissions interfere with each other according to the interference model (cf. section 3).

The Two-Phase algorithm (see algorithm 1) repeatedly assigns colours to the vertices of the interference graph in such a way that two adjacent vertices do not have any colour in common. The iteration of colourings is divided into two phases. There is a certain analogy to water filling, as resources are divided equally among flows, and bottlenecks are identified and the affected flows blocked in the second phase. The throughput of each flow is determined by dividing its number of colourings by the total number of used colours.

The first phase of the algorithm provides all flows with an equal throughput. In the beginning, the number of new colours (which are introduced only if necessary) decreases with each iteration due to the rising number of already used colours. After further iterations, a point is reached where the number of new colours stagnates at a certain value or within a small interval, indicating a region in the interference graph which is hard to colour (e.g. a large clique). Such regions represent bottlenecks. With regard to possible outliers in early iterations, the phase change takes place only if stagnation has been detected several times.

In the second phase, the remaining unused resources are distributed among the flows. The restricted availability of colours can lead to *conflicts* between flows in an iteration, e.g. when the number of available colours does not suffice to colour all flow links sharing the

Algorithm 1 Two-Phase colouring algorithm

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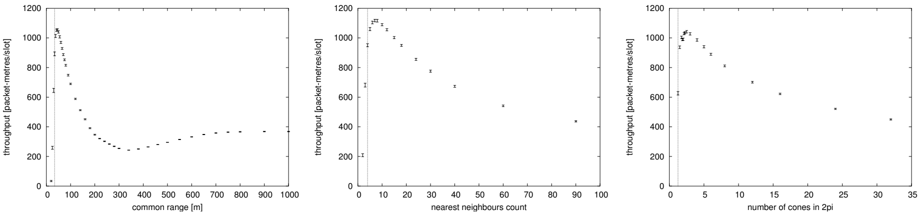
Input: Interference graph $G = (V, E)$, flow mapping $m : V \to F$, threshold value $t \in \mathbf{N}$
  $c_{\text{old}} := \infty$
  WHILE $t > 0$ DO {first phase}
    FOR ALL $v \in V$ DO
      assign another colour to $v$ (use a new colour if and only if necessary)
    END FOR
    $c_{\text{new}}$ := number of new colours (not used in any preceding iteration)
    IF $c_{\text{new}} \geq c_{\text{old}}$ THEN
      $t := t - 1$
    END IF
    $c_{\text{old}} := c_{\text{new}}$
  END WHILE
  WHILE $F \neq \emptyset$ DO {second phase}
    FOR ALL $f \in F$ DO
      IF all $v \in V$ with $m(v) = f$ can be coloured with colours already used THEN
        colour all $v \in V$ with $m(v) = f$ with these colours
      ELSE
        $F := F \setminus \{f\}$
      END IF
    END FOR
  END WHILE
```

same physical link. So, flows with equal prerequisites may obtain different throughput, violating the max-min fairness constraint. Unfortunately, although detecting the example conflict is simple, this is very difficult in general, making a perfectly max-min fair solution infeasible. But as the number of delivered packets of conflicting flows differs by one at most, the deviation from max-min fairness is negligible, if the first phase lasts long enough.

5 Results

Figure 1 shows the mean overall throughput (weighted by the stream distances) of 250 streams for different power control strategies with varying parameter settings in networks with 1000 stations uniformly distibuted on a square area with 1000m along the diagonal. The values conform very good to the simulation results presented in [GdWFM04], while requiring a much shorter computing time. Our implementation of Two-Phase uses the well-known greedy heuristic *Saturation Largest First*, proposed as DSATUR in [Br79]. In [Ra97] it is shown that heuristic colouring provides good practical results on interference graphs. This is confirmed by the higher overall throughput calculated by our algorithm. The colouring heuristic is able to find good independent transmission sets, whereas the simulation uses a rather simple strategy to determine simultaneous transmissions.



(a) Common ranges (b) Nearest Neighbours (c) Cone-based

Figure 1: Mean overall throughput for different power control strategies

6 Conclusion & Future Work

In this paper, we have introduced an efficient method for evaluating the transport capacity of wireless multihop network topologies. We consider this as a link between theoretical and simulative analysis of the capacity of ad hoc networks. Future work might concentrate on relaxing the requirement of predefined routes. Instead, it would be useful to evaluate topologies based on multipath routing as in [GdWFM04], or even based on probabilities for routes to be established.

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