A mathematical model to quantitatively calculate the trade-offs between ESs within a DSS

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Abstract: Farmers tend to develop a production plan for their farm based on experience and expectations regarding costs, yields and revenues for their different production options. This is – especially in connection with livestock and/or biogas production systems – a complex decision problem that can be solved using mathematical programming approaches. Currently farmers are additionally expected to consider ecosystems services when optimising their production, some of which can be rewarded through various payment schemes. The study of the trade-off between ESs is an important issue that helps to optimize decisions about their use as well as their conservation. Despite the increase in the number of studies on ESs and the trade-off between them, the number of studies on how to quantitatively calculate and manage these trade-offs is still scarce. In the absence of a market to directly discover the value of ESs, economic valuation is a useful research and policy tool for determining the value and prioritization of ESs by assigning a monetary value for the services provided by them. But these methods usually value ESs separately and based on surveys of their consumers' willingness to pay. The whole farm mathematical modelling approach within the project DAKIS (Digital Agricultural Knowledge and Information System) will allow for the determination of the shadow value of ESs simultaneously based on their society's supply and demand and their impact on the farmer's income or cost. This conceptual paper, while introducing this feature in order to evaluate and prioritize the ESs more comprehensively and accurately, shows how it can be used to quantify the effect of changes in the society's supply and demand of each ESs on the trade-offs between them.

Keywords: ESs trade-off, economic valuation, shadow value, Decision Support System, DAKIS

1 Introduction

Increasing environmental awareness, along with the implementation of incentive policies by governments in this regard, has made it important to focus on ESs in designing DSS which are supposed to help farmers in determining their optimal copping pattern. But the challenge is how to value and prioritize ESs in optimizations while there is always trade-off between them [Li21]. Although many studies have been conducted by researchers on the trade-off between ESs, only a limited number of studies have been conducted about quantifying this trade-off [Na20]. In the absence of a market for pricing ESs, economic valuations that assign a monetary value to ESs are good tools to prioritize ESs [MKO19]. However, these tools have two main problems. First, despite the existence of an obvious trade-off between ESs, they are valued separately. Second, these evaluations are based

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solely on the opinions of their consumers or producers about their willingness to pay or to receive. Therefore, in order to eliminate these weaknesses, first, it is necessary to evaluate the ESs simultaneously and second, this valuation should be based on the actual income that their beneficiaries earn from their consumption or the actual cost that their producers expense to produce them. The whole farm mathematical modelling approach within the project DAKIS is part of a chain of measurements and models using new technologies to determine the level of ESs on farm consumption and provision for specific (soil, weather, spatial context) cropping systems. The whole farm model will obtain this information as additional coefficients for each production activity. It is based on MODAM[2] [Sc20]; [Vi18], to which equations have been added for valuation of ESs and which is used to determine the optimal amount of their provision and consumption within a farm.

2 General form of whole farm mathematical model within DAKIS

The general simplified form of the mathematical model which will be used within DAKIS is as follows:

$$\underset{X}{\text{Max}} \quad GM = \sum_{t=1}^{T} \sum_{f=1}^{F} \sum_{j=1}^{J} (1+r)^{-t} gm_{t,j} X_{t,f,j}$$

$$\sum_{f=1}^{F} \sum_{j=1}^{J} a_{t,i,f,j} X_{t,f,j} \leq b_{t,i} \qquad for\ t = 1,2,\dots,T\ and\ i = 1,2,\dots,I$$

$$\sum_{f=1}^{F} \sum_{j=1}^{J} c_{t,s,f,j} X_{t,f,j} \leq ess_{t,s} \qquad for\ t = 1,2,\dots,T\ and\ s = 1,2,\dots,S$$

$$\sum_{f=1}^{F} \sum_{j=1}^{J} p_{t,d,f,j} X_{t,f,j} \geq esd_{t,d} \qquad for\ t = 1,2,\dots,T\ and\ d = 1,2,\dots,D$$

$$X_{t,f,j} \geq 0 \qquad (1)$$

Where GM is the maximized net present value of farmer's total gross margin during the planning years, r is discount rate, $gm_{t,j}$ is the gross margin of one unit of crop *j* in year *t*, $X_{t,f,j}$ is the optimal cultivation area of crop *j* at the field *f* in year *t*, $a_{t,i,f,j}$ is technical coefficient of constraint *i* for producing one unit of crop *j* at field *f* in year *t*, $b_{t,i}$ is total available amount of constraint *i* in year *t*, $c_{t,s,f,j}$ is consumption of ESs *s* for producing one unit of crop *j* at field *f* in year *t* e.g. soil nitrogen content as a result of producing one hectare of each crop, $ess_{t,s}$ is total society's supply of ESs *s* in year *t* e.g. maximum allowable Nitrate concentration in leachate, $p_{t,d,f,j}$ is provision of ESs *d* by producing one unit of crop *j* at field *f* in year *t* e.g. erosion-reducing effect of producing one hectare of each crop and $esd_{t,d}$ is total society's demand of ESs *d* in year *t* e.g. recommended minimum erosion control. It also should be noted that a whole farm model has many components including Herd, Feeding, Stables, Buildings, Investments, Machinery, Labour, Manure, Fertilization, Biogas components, where the first constraint $b_{t,i}$ of the

[2] Multi-Objective Decision support tool for Agro-ecosystem Management

model represents all these components. Taking into account the slack variable of constraint *i* in year *t* ($Slack^{b}_{t,i}$), slack variable of society's supply of ESs *s* in year *t* ($Slack^{ess}_{t,s}$) and surplus variable of society's demand of ESs *d* in year *t* ($Surplus^{esd}_{t,d}$) in order to convert the constraints with unequal sign to equal, the above model can be converted to the following Lagrange function.

$$\max_{X} \quad \mathcal{L} = \sum_{t=1}^{T}\sum_{f=1}^{F}\sum_{j=1}^{J}(1+r)^{-t} gm_{t,j}X_{t,f,j} + \sum_{t=1}^{T}\sum_{i=1}^{I}\Lambda_{t,i}\big[b_{t,i} - \sum_{f=1}^{F}\sum_{j=1}^{J}a_{t,i,f,j}X_{t,f,j} - Slack^{b}_{t,i}\big] + \sum_{t=1}^{T}\sum_{s=1}^{S}SR_{t,s}\big[ess_{t,s} - \sum_{f=1}^{F}\sum_{j=1}^{J}c_{t,s,f,j}X_{t,f,j} - Slack^{ess}_{t,s}\big] + \sum_{t=1}^{T}\sum_{d=1}^{D}SC_{t,d}\big[esd_{t,d} - \sum_{f=1}^{F}\sum_{j=1}^{J}p_{t,d,f,j}X_{t,f,j} + Surplus^{esd}_{t,d}\big] \quad (2)$$

Where, Lagrange multipliers $\Lambda_{t,i}$, $SR_{t,s}$ and $SC_{t,d}$ are dual or shadow prices of $b_{t,i}$, $ess_{t,s}$ and $esd_{t,d}$ respectively and can be defined as follows.

$$\Lambda_{t,i} = {}^{\partial\mathcal{L}}/_{\partial b_{t,i}} \quad , \quad SR_{t,s} = {}^{\partial\mathcal{L}}/_{\partial ess_{t,s}} \quad , \quad SC_{t,d} = {}^{\partial\mathcal{L}}/_{\partial esd_{t,d}} \quad (3)$$

$SR_{t,s}$ calculates the change of farmer's gross margin as a result of consumption of one more unit of ESs *s* in year *t* byvf farmer "*Ceteris paribus*". Also, $SC_{t,d}$ measures the change of farmer's gross margin as a result of provision of one more unit of ESs *d* in year *t* by farmer "*Ceteris paribus*". As can be seen, first, these values, regardless of the farmer's willingness to pay or receive, are calculated on the basis of the income generated by the consumption of each unit of ESs for the farmer or the cost incurred by the farmer in producing each unit of ESs. Second, as these values are calculated simultaneously, the interaction and trade-off between ESs are taken into account in their measurement. To calculate these shadow prices directly, the following dual model can be solved.

$$\min_{\Lambda,SR,SC} \quad \sum_{t=1}^{T}\sum_{i=1}^{I}b_{t,i}\Lambda_{t,i} + \sum_{t=1}^{T}\sum_{s=1}^{S}ess_{t,s}SR_{t,s} + \sum_{t=1}^{T}\sum_{d=1}^{D}esd_{t,d}SC_{t,d}$$

$$\sum_{i=1}^{I}a_{t,i,f,j}\Lambda_{t,i} + \sum_{s=1}^{S}c_{t,s,f,j}SR_{t,s} + \sum_{d=1}^{D}p_{t,d,f,j}SC_{t,d} \geq (1+r)^{-t}gm_{t,j}$$

$$for\ t = 1,2,\dots,T, f = 1,2,\dots,F\ and\ j = 1,2,\dots,J$$

$$\Lambda_{t,i} \geq 0, SR_{t,s} \geq 0\ and\ SC_{t,d} \leq 0 \quad (4)$$

$SR_{t,s}$ is a good measure to calculate the Green Tax on ESs that the farmer consumes and $SC_{t,d}$ is an appropriate criterion to determine the Payments for ESs that the farmer produces. The point to be noted in solving Model (1) or its dual Model (4) is that, since the purpose of solving them is to simultaneously evaluate the ESs by calculating the shadow prices of their society's supply and demand constraints, this model should not be nonlinear or integer in any way. Therefore, it is necessary to temporarily exclude binary decision variables that will possibly enter the model for example to select different investment options (and not all investment related equations). These variables will be added again later to the final model (see section 4), which does not need to be linear or non-integer.

3 Quantification of trade-off between ESs

Of course, the level of these shadow prices depends entirely on the society's supply and demand of each ESs. This dependence can be quantified by performing a sensitivity analysis on the society's supply and demand of each ESs. Performing this sensitivity analysis, in addition to determining the shadow price of ESs in each range of society's supply and demand, will also determine the shadow price of other ESs in those ranges and in this way, quantification of trade-off between ESs will be achieved. For example if the shadow return of ESs *s* in year *t* is $SR^{*}_{t,s}$, this value will be valid at society's supply levels $ess^{min}_{t,s}$ and $ess^{max}_{t,s}$ of the ESs. Increasing the society's supply from $ess^{max}_{t,s}$ will reduce the shadow price to $SR^{**}_{t,s}$ and decreasing the society's supply from $ess^{min}_{t,s}$ will increase the shadow price to $SR^{***}_{t,s}$. By performing the society's supply sensitivity analysis of the ESs *s* in year *t* continuously, its shadow price for each range of society's supply can be calculated in the form of Table 1. The result of these calculations will be the deriving of the on farm demand function for ESs *s* in year *t* as shown in Figure 1.

Range of feasibility of society's supply of ESs *s* in year *t*	Shadow Return of ESs *s* in year *t*
$0 \leq ess_{t,s} < ess^{nmin}_{t,s}$	$SR^{****}_{t,s}$
$ess^{nmin}_{t,s} \leq ess_{t,s} < ess^{min}_{t,s}$	$SR^{***}_{t,s}$
$ess^{min}_{t,s} \leq ess_{t,s} \leq ess^{max}_{t,s}$	$SR^{*}_{t,s}$
$ess^{max}_{t,s} < ess_{t,s}$	$SR^{**}_{t,s}$
$ess^{nmax}_{t,s} < ess_{t,s} \leq \infty$ ^{nmax}	0

Tab. 1: Sensitivity analysis of society's supply of ESs *s* in year *t*

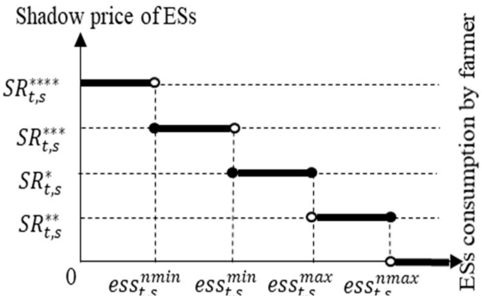

Fig. 1: On farm demand function for ESs *s* in year *t*

Similarly, the society's demand for each ESs can be sensitively analysed and the shadow prices corresponding to each range of society's demand can be calculated. For example, if the shadow cost of ESs *d* in year *t* in the range of society's demand $esd^{min}_{t,d}$ and $esd^{max}_{t,d}$ is equal to $SC^{*}_{t,d}$, a rise in society's demand from $esd^{max}_{t,d}$ causes a rise in shadow price to the level $SC^{***}_{t,d}$ and a drop in society's demand from level $esd^{min}_{t,d}$, causes a fall in shadow price to the level $SC^{**}_{t,d}$. Continuing this sensitivity analysis, the shadow price of ESs d in year *t* for each range of society's demand is specified in Table 2. Using this information, on farm supply function of ESs *d* in year *t* can be derived as shown in Figure 2.

Range of feasibility of society's demand of ESs *d* in year *t*	Shadow cost of ESs *d* in year *t*
$0 \leq esd_{t,d} < esd_{t,d}^{nmin}$	0
$esd_{t,d}^{nmin} \leq esd_{t,d} < esd_{t,d}^{min}$	$SC_{t,d}^{**}$
$esd_{t,d}^{min} \leq esd_{t,d} \leq esd_{t,d}^{max}$	$SC_{t,d}^{*}$
$esd_{t,d}^{max} < esd_{t,d} \leq esd_{t,d}^{nmax}$	$SC_{t,d}^{***}$
$esd_{t,d}^{nmax} < esd_{t,d} \leq \infty$	$SC_{t,d}^{****}$

Tab. 2: Sensitivity analysis of society's demand of ESs *d* in year *t*

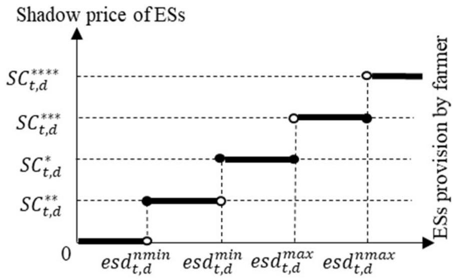

Fig. 2: On farm supply function for ESs *d* in year *t*

4 Determination of optimal consumption and provision of ESs

Having the shadow price of each ESs and using them as the weights of each ESs (Optimal values of *SR* and *SC* obtained from solving Model (4)), it will be possible to determine the optimal amount of consumption ($ESS_{t,s}^{c}$) and provision ($ESS_{t,d}^{p}$) of each ESs by the farmer at the same time as optimizing the whole farm activities. For this purpose, as shown in the following model, instead of maximizing the farmer's gross margin, the social gross margin which in addition to explicit economic returns also includes implicit incomes and costs from consumption and provision of ESs is maximized.

$$\underset{X,ESS^c,ESS^p}{\text{Max}} \quad SGM = \sum_{t=1}^{T}\sum_{f=1}^{F}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{d=1}^{D}(1+r)^{-t}\left[gm_{t,j}X_{t,f,j} - \overline{SR}_{t,s}ESS_{t,s}^{c} - \overline{SC}_{t,d}ESS_{t,d}^{p}\right]$$

$$\sum_{f=1}^{F}\sum_{j=1}^{J} a_{t,i,f,j}X_{t,f,j} \leq b_{t,i} \qquad for\ t = 1,2,\dots,T\ and\ i = 1,2,\dots,I$$

$$\sum_{f=1}^{F}\sum_{j=1}^{J} c_{t,s,f,j}X_{t,f,j} = ESS_{t,s}^{c} \qquad for\ t = 1,2,\dots,T\ and\ s = 1,2,\dots,S$$

$$\sum_{f=1}^{F}\sum_{j=1}^{J} p_{t,d,f,j}X_{t,f,j} = ESS_{t,d}^{p} \qquad for\ t = 1,2,\dots,T\ and\ d = 1,2,\dots,D$$

$$ESS_{t,s}^{c} \leq ess_{t,s} \qquad for\ t = 1,2,\dots,T\ and\ s = 1,2,\dots,S$$

$$ESS_{t,d}^{p} \geq esd_{t,d} \qquad for\ t = 1,2,\dots,T\ and\ d = 1,2,\dots,D$$

$$X_{t,f,j} \geq 0, ESS_{t,s}^{c} \geq 0\ and\ ESS_{t,d}^{p} \geq 0 \tag{5}$$

In order to calculate the social gross margin (SGM), it is sufficient to subtract the social cost of consuming ESs that generates private income for the farmer from her/his gross margin and add the social income of ESs provision that creates private costs for the farmer to her her/his gross margin. To calculate the social cost and income of consumption and provision of ESs, respectively the shadow return ($\overline{SR}_{t,s}$) and shadow cost ($\overline{SC}_{t,d}$) of each unit of ESs, which have already been calculated as their weights, are used. In other words, this model measures the optimal level of crops production as well as the optimal level of the provision and consumption of ESs, provided that the total social cost of consuming ESs is deducted from farmer's gross margin as a green tax and the total social income from the provision of ESs is given to her/him as a payment for ESs.

5 Conclusion

Due to the need to considering ESs in designing agricultural DSS on the one hand and the problem of valuation and prioritization of ESs in these DSS due to the existence of trade-off between them on the other hand, this conceptual paper introduces the whole farm mathematical modelling approach within DAKIS which has the ability to address this challenge. In this model, by simultaneously valuing ESs by calculating their shadow price, the optimal cropping pattern of farmers is determined by maximizing their social gross margin instead of private gross margin. Also, using this model, it will be possible to derive the on farm supply and demand functions for each ESs and examine the effect of their changes on the trade-off between ESs.

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