Consensus and Relation Networks

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Abstract: In this paper, we propose an extension to bounded confidence model which is a nonlinear opinion dynamics model. The goal of this new extension is to emphasize the underlying network structure of the model, since in real life network structure plays an important , if not vital, role in opinion dynamics. And with simulation results, we show how networks, such as grid and scale-free, affect the consensus formation in bounded confidence model separately as well as together.

1 Introduction

An opinion is a subjective statement or thought about an issue or a topic. Among a group of individuals, through interaction, they may change their opinions and eventually the dynamics may lead to fragmentation, polarization or consensus. Bounded Confidence (BC) model is one of the efforts to model this dynamic process, and it is first proposed by Krause in 1997 [KS97]. Later in 2002 Hegselmann and Krause [HK02] studied the model both in historical and analytical details, and discussed its simulation results. They also have mentioned the preliminary results for taking the network structures into account in the model, in particular grid network. In 2005 Hegselmann and Krause [HK05] further discussed the effect of different averaging methods, such as arithmetic mean, geometric mean, power mean as well as random mean, in opinion aggregation. In 2000 Deffuant et al. [De00] proposed another opinion dynamics model, while it is similar to the BC model by Krause and Hegselmann, the difference between the them is the opinion exchange process. Till now BC model has been studied from different angles and approaches [Fo05a] , [Fo05b], [Fo05c], [HK06], [ULH08], [BHT09]. Among them, Fortunato in [Fo05a] has shown that network structures do matter in a BC model. The different network structures used in the demonstration are grid, random graph, scale-free graph and complete graph. With simulation, the consensus threshold for those graphs are shown and discussed.

Those works investigate opinion dynamics by analytical methods as well as by computer simulations, considering only one network structure at a time. In the real life scenario, networks are much more complex than that, specially in our era with the fast development of artificial social networks. People interact, communicate and share opinions through different networks of various social influences such as family relations, social network relations and professional relations, to name a few. In order to better imitate this point, in this paper we propose to use two networks at the same time, and show how it influences the consensus threshold of BC model via multi-agent simulation.

The paper is organized as the following: section two introduces opinion dynamics models, in particular BC model. Then in section three, grid and scale free graphs are introduced as they will be used later in the model. In section four our extension of BC model is introduced and simulation results are discussed. Finally, the paper is closed with a conclusion.

2 Opinion Dynamics

Early formulation of opinion dynamics was given by J.R.P. French in 1956 [Fr56] in order to understand complex phenomena found empirically about groups. This work was followed by M.H. De Groot in 1974 [Gr74] and by K. Lehrer in 1975 [Le75]. In general, they deal with simple cellular automata, where people become the vertices of a graph and the neighboring vertices represent agents which have a personal relationship (acquaintance). A simple rule determines how the opinion of an agent is influenced from (or can influence) that of its neighbors. The aim is to understand how it happens that the large groups of people ultimately share the same opinion, starting from a situation in which everybody has its own ideas independently of those people with whom they interact. As difficult as these early models might be in their details, they are all comparatively simple in the sense that they are all linear models i.e. the structure of the model don't changes with the states of the model given by the opinions of the agents. The first nonlinear model was formulated and analyzed in [KS97][Kr00], in these works Krause introduced also bounded confidence, which describes the fact that peers holding opinions that are sufficiently different from an agent's own opinion do not exhibit any influence on this agent. On this basis, Hegselmann & Krause [HK02] introduced a model of continuous opinion dynamics where agents perceive all other agents' opinions with bounded confidence.

2.1 Bounded Confidence Model

Bounded confidence model is also called Hegselmann & Krause (HK) Model. It is a continuous opinion dynamics model with bounded confidence.This means it is a model where the opinions are real numbers between 0 and 1, and two agents are compatible for interaction if the difference of their opinions is smaller than the confidence bound parameter ϵ.

Let n be the number of agents in the group under consideration. To model the repeated process of opinion formation we think of time as a number or rounds of periods, that is as discrete time $T = \{0, 1, 2, \ldots\}$. It is assumed that the opinion of an agent is continuous and expressed by a real number. For a fixed agent, say i where $1 \leq i \leq n$, with n denoting the number of agents, the agent's opinion at time t is $x_i(t) \in [0..1]$. Thus $x_i(t)$ is a real number and the vector $x(t) = (x_1(t), \ldots, x_n(t))$ in $n - dimensional$ space represents the opinion profile of the system at time t. Fixing an agent i, the weight given to any other agent, say j, is denoted by a_{ij} with $a_{i1} + a_{i2} + \ldots + a_{in} = 1$ and $a_{ij} \geq 0$ for all i, j. Having these notations, opinion formation of agent i can be described as averaging in the following way

$$x_i(t+1) = a_{i1}x_1(t) + a_{i2}x_2(t) + \ldots + a_{in}x_n(t) \tag{1}$$

That is, agent i adjusts his opinion in period $t+1$ by taking a weighted average with weight a_{ij} for the opinion of agent j, $1 \leq j \leq n$, at time t. Of course, weights can be zero. For example, if agent i disregards all other opinions, this means $a_{ii} = 1$ and $a_{ij} = 0$ for $j \neq i$ or, if i follows the opinion of j then $a_{ij} = 1$ and $a_{ik} = 0$ for $k \neq j$. It is important to note that the weights may change with time or with the opinion, that is $a_{ij} = a_{ij}(t, x(t))$ can be a function of t and/or of the whole profile vector $x(t)$. By collecting the weights into a matrix, $A(t, x(t)) = (a_{ij}(t, x(t)))$, with n rows and n columns, we obtain a stochastic matrix, i.e., a nonnegative matrix with all its rows summing up to 1. Thus, using matrix notation, the general form of the HK model can be compactly written as

$$x(t+1) = A(t, x(t))x(t) \quad for\, t \in T \tag{2}$$

The HK model portrays bounded confidence among the agents in the following sense. An agent i takes only those agents j into account whose opinions differ from his own not more than a certain confidence level ϵ. Fixing an agent i and an opinion profile $x = (x_1, \ldots, x_n)$ this set of agents is given by

$$I(i, x) = \{1 \leq j \leq n \;\; with \;\; |x_i - x_j| \leq \epsilon_i\} \tag{3}$$

Thus the model with bounded confidence is given by

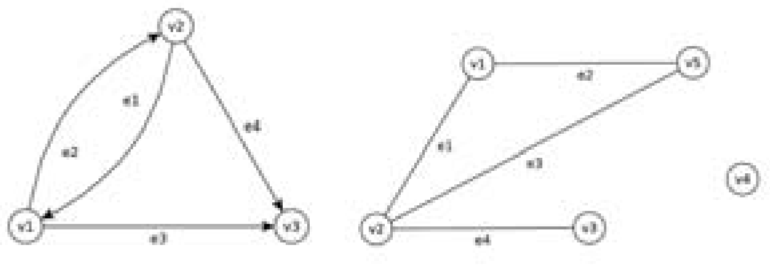
$$x_i(t+1) = |I(i, x(t))|^{-1} \sum x_j(t) \quad for\, t \in T \tag{4}$$

Hegselmann and Krause have also explored their model with different configuration of ϵ [HK02]. For example, symmetric and asymmetric confidence interval, opinion dependent and independent asymmetric confidence interval, etc.

3 Graph Representation of Networks

In the most common sense of the term, a graph is an ordered pair $G := (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or lines which are 2-element subsets of V (see Figure 1).

In modeling opinion dynamics with agents and graphs, agents will be represented by vertices and communications between agents by links between vertices. For we suppose that the interaction between agents be mutual, undirected graphs are used in our model.



(a) (b)

Figure 1: Graph examples: (a) a directed graph; (b) an undirected graph.

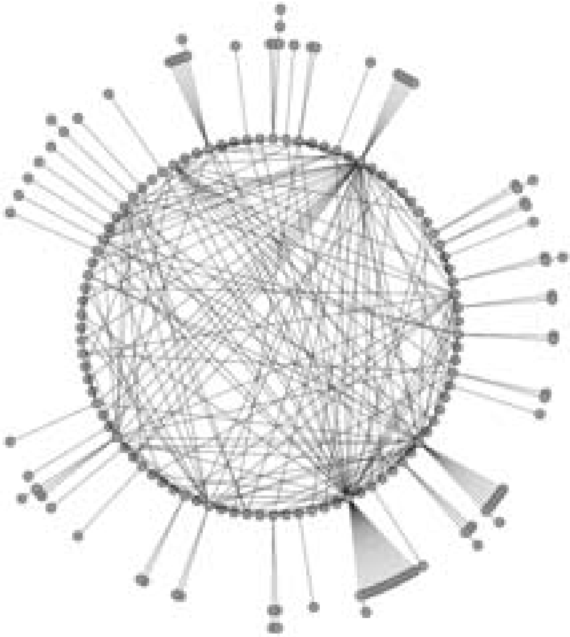
3.1 Scale-free Graph

A scale-free graph is a graph whose degree distribution follows a power law. A key ingredient in scale free network is preferential attachment, i.e., the assumption that the likelihood of receiving new edges increases with the nodeâs degree. The Barabási-Albert model assumes that the probability $P(k)$ that a node attaches to node i is proportional to the degree k of node i, that is $P(k) \approx k^{-\gamma}$ where γ is a constant whose value is typically in the range $2 < \gamma < 3$, although occasionally it may lie outside these bounds. Figure 2 shows a scale free graph and its degree distribution [1]. Scale-free graphs are noteworthy because many empirically observed networks appear to be scale-free, including protein networks, citation networks, some social networks [AB02] and the world wide web as well. For those reasons scale free network is chosen in our model to represent agents' communication by means of social networks, mail or other internet tools.

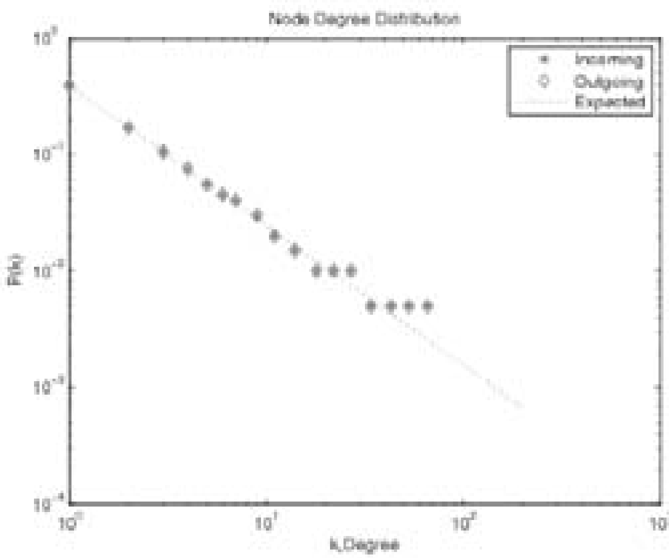
3.2 Grid Graph

Grid or lattice graph refer to a number of categories of graphs whose drawing corresponds to some grid/mesh/lattice, i.e., its vertices correspond to the nodes of the mesh and its edges correspond to the ties between the vertices. So, a grid graph is a unit distance graph corresponding to the square lattice, so that it is isomorphic to the graph having a vertex corresponding to every pair of integers (a, b), and an edge connecting (a, b) to $(a + 1, b)$ and $(a, b + 1)$. The finite grid graph $G_{m,n}$ is an $m \times n$ rectangular graph isomorphic to the one obtained by restricting the ordered pairs to the range $0 \leq a < m$, $0 \leq b < n$. Grid graphs can be obtained as the Cartesian product of two paths: $G_{m,n} = P_m \times P_n$. Grid is very important in agent based models, notably in cellular automata, to model local relationship between agents. Normally grid is used with a periodic boundary condition to simulate large systems by modeling a small part, which is the case in our simulation, too.

[1] The graph is created using Lev Muchnik's Complex Networks package for matlab: http://www.levmuchnik.net/Content/Networks/ComplexNetworksPackage.html. Visited in April, 2011.



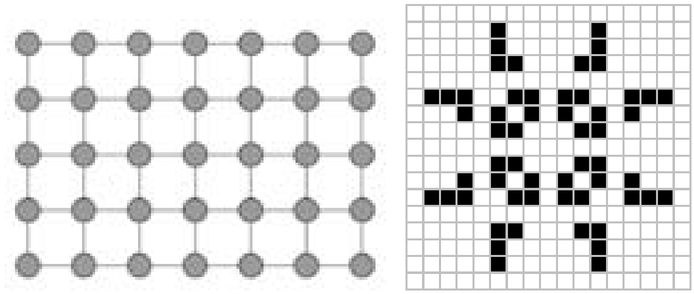
(a)



(b)

Figure 2: (a) A scale free graph. (b) Degree distribution of graph (a).

Figure 3 shows a grid graph, and Conway's game of life as an example of periodic grid's usage in modeling.



(a) (b)

Figure 3: (a) A grid graph with $n = 5$ and $m = 7$. (b) Conway's game of life.

4 The Model and Simulation Results

As we have seen in the previous section, in modeling, grid networks emphasize local neighborhood interactions, while scale-free networks highlight those not geographically restricted, say internet or social networks. To imitate as close as possible real life interactions, we propose to integrate those two networks at the same time in BC model. We also study their impact on the behavior of the consensus threshold through simulation.

4.1 The Coupled Network Model

As said previously, our model is based on mixing two BC opinion dynamics models, one using a grid graph as the relation network and the second using a scale-free graph as following:

- Number of agents: $N = \{0, 1, 2, \ldots, n\}$
- Discrete time: $T = \{0, 1, 2, \ldots\}$
- Opinion formation:

$$x_i(t+1) = |I(i, x(t))|^{-1} \sum x_j(t) + |J(i, x(t))|^{-1} \sum x_j(t) \tag{5}$$

with:

- For the grid graph

$$I(i,x) = \{1 \leq j \leq n \; with \; |x_i - x_j| \leq \epsilon_{1,i}\} \tag{6}$$

- For the scale-free graph

$$J(i,x) = \{1 \leq j \leq n \; with \; |x_i - x_j| \leq \epsilon_{2,i}\} \tag{7}$$

4.2 Simulation Results

The simulation is realized with MultiAgent simulation platform Repast. We first give the Results for Grid network, and Scale-Free network, finally the results of mixed network. In Figure 4, we can see a screenshot from simulation. The different opinions are colored differently, and each cell in the grid are connected to its 4 neighbors, as well as through scale-free network to other possibly not local cells, whose edges are represented with gray links. In judging the consensus we use the notion of consensus probability, which is calculated by dividing the number of agents in the largest cluster by the total number of agents in the system. Of course, this is calculated when the simulation is stabilized, which means there will occur no more *opinion changes* in the simulation. Our criterion for "*no opinion changes*" is to check whether any opinion varied by less than 10^{-9} after a time step.

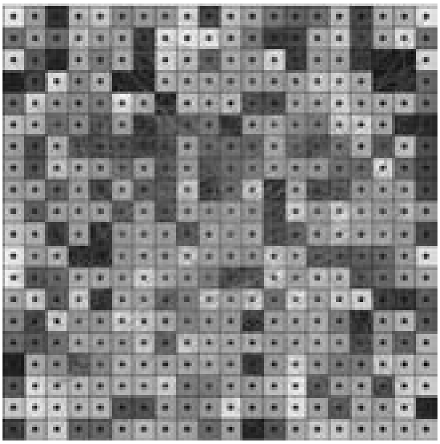


Figure 4: A peek at the simulation

4.2.1 BC Model with Grid Network

In simulation we tested the grid network BC model with 400, 900, 1600 and 6400 nodes respectively. The bounded confidence variable ϵ is ranged from 0.2 to 0.6, increasing 0.01 each step.

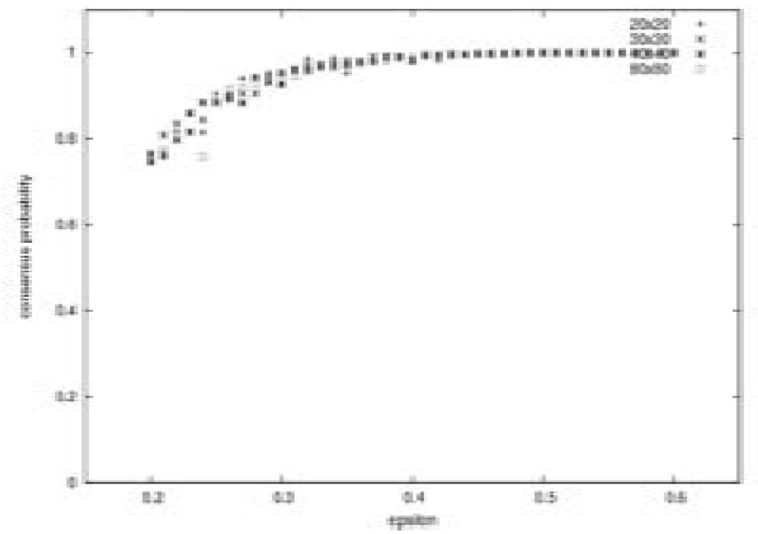


Figure 5: BC model on periodic grid networks

From Figure 5, we can see that starting from $\epsilon = 0.4$ the system achieves almost consensus disregard of the agent number. In the simulation we also noticed that the increasing ϵ shortens the time needed for stabilization. Besides, with the growing number of nodes system needs more time to stabilize.

4.2.2 BC Model with Scale-free Network

In this simulation, scale-free networks with 400,900,1600 and 2500 nodes are tested. The Barabási–Albert scale-free network is used [AB02]. It is known that this method results a network which has a degree distribution with a power law tail, and exponent of the power law is 3. In this simulations, ϵ is varied from 0.2 to 0.6, in the same way as above.

From Figure 6 we can see while epsilon is changing from 0.2 to 0.6, the consensus probability will slightly improve from around 0.4 to around 0.6. We also investigated with bounded confidence of 1, simulation still stabilizes around 0.6, no full consensus is achieved. This is because the average degree of the networks that is used here equals 1. Thus it is inevitable to have isolated agents. Contrary to the BC model with grid network, it seems that the increasing number of nodes does not evidently delay the stabilization of the system. This is again due to the particularity of the scale-free networks. We need to verify this with more simulations and further analysis.

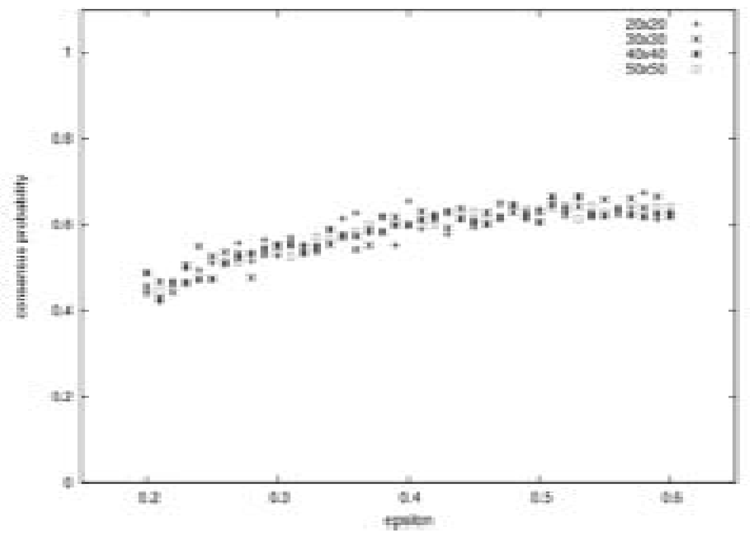


Figure 6: BC model on BA scale-free networks

4.2.3 BC Model with Grid and Scale-free Networks

Here is the most interesting part of the simulation, since it shows the effect of coupled network structures on the consensus threshold. Even if we have tested this model with varying number of nodes, here we only show the ones with 900 and 2500 nodes. We denote the bounded confidence for grid network as ϵ_1, and denote that of scale-free network as ϵ_2. As shown previously, $\epsilon_{1,2} = [0.1..0.5]$, with an incremental step of 0.05.

In Figure 7 and 8, the consensus threshold space is shown. Since scale-free network structure fails to achieve a consensus alone, when $\epsilon_1 = 0.1$, the increase of ϵ_2 value again does not result a consensus. If we change the confidence interval in grid network ϵ_1 to 0.2, by increasing ϵ_2, simulation results a consensus state. Despite of the ϵ_2, increasing ϵ_1 always results a consensus. However, consensus threshold of ϵ_1 changes according to the value of ϵ_2. We can conclude that while those two networks compensate each other to some extent in achieving consensus, the stronger one plays a more important role, as in the case of grid network. Meanwhile, the role of scale-free network is still notable in that it not only accelerates the consensus process, but also it helps to achieve consensus when the grid confidence is not high enough.

As we can see in two previous cases, curves are identical despite of the increasing number of nodes. However, we noticed that the time to achieve stabilization in the system increases along with the number of nodes, clearly a direct consequence of underlying grid network, though not so evident as in the case of grid only simulation.

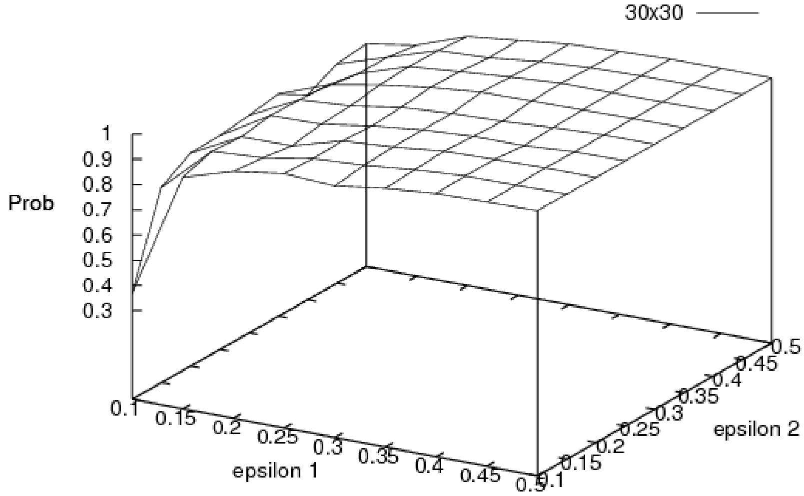


Figure 7: BC model on a mixed network with 900 agents

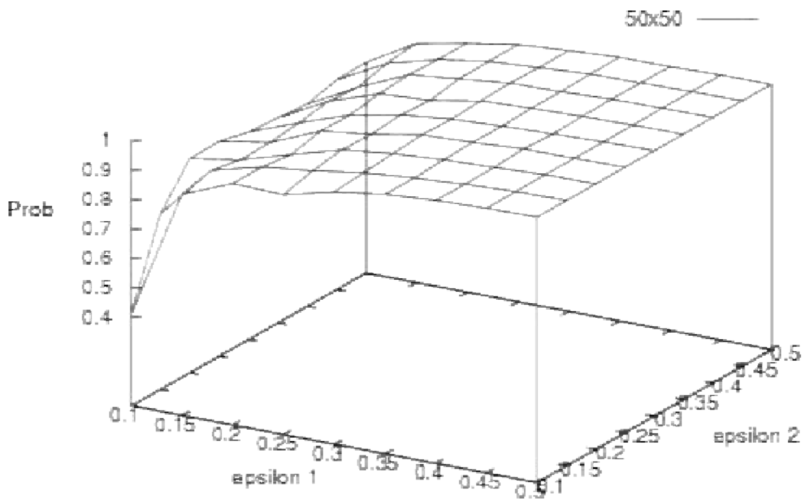


Figure 8: BC model on a mixed network with 2500 agents

5 Conclusion

Although BC model has been studied heavily among researchers and some of those works have studied the relationship between underlying network structure and consensus threshold, little work has been done on integrating several network structures into a single model. The important part of our model is that we introduced multi-relationship in the opinion dynamics. Despite the fact that only two networks are considered at the first step, this work has shown the possibilities of achieving more complex and realistic network structures by integrating multiple networks. Furthermore the preliminary simulations have shown us that consensus threshold for BC model changes according to the multi-network structures.

Possible future works that we are considering will continue in this direction. These results encourage us to further explore the multi-network structures as a underlying structure of the opinion dynamics models. Besides, we would also like to study the dynamic networks as the result of opinion dynamics.

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