

# A Signature Complexity Measure to select Reference Signatures for Online Signature Verification

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**Abstract:** This paper presents an original procedure for selecting the reference online signature instances of a writer, an important issue for any effective signature verifier. To this end, for each signature instance, we propose a novel complexity measure, by exploiting a global description of signatures in the frequency domain as well as a global statistical modelling of each signature instance. To select the reference signatures, we propose a method based on the distribution of complexity values for all the available genuine signatures. The 2500 genuine samples of MCYT-100 online database are used in this study. Experimental results show the effectiveness of the method and of the here proposed complexity measure for this specific task.

## 1 Introduction

One of the most widespread means to verify the identity of a person in our society is handwritten signature, and that since a long time, for example as a mean of guaranteeing the validity of a document in the legal field or in banking transactions [HG14]. Nowadays, signatures can either be acquired online as a temporal signal on a digitizer or a smartphone [HG14, IP08] or offline as a static image [IP08, SPL92]. Our study is carried out in the online framework.

The implementation of an automatic signature verification system consists of two phases: enrolment and verification. The enrolment consists in the acquisition of signatures that will be stored as references or be used to build a writer-model. During verification, the writer claims an identity and captures his/her probe signature, then given as input to the verification system; its outcome is the acceptance or rejection of the writer's claimed identity. Thus, enrolment is the first step of any verifier, and is a crucial phase for improving the reliability of the verification system [BP89, Di99].

It is well-known that signature is a behavioral biometric modality with high intra-class variability. Such variability is the main obstacle for accurate signature verification. For

this reason, it is essential to have an effective criterion for selecting the pertinent signatures of the reference set. Most previous works on handwritten signature are focused on the verification step, while only very few tackle the selection of reference signatures based on signature stability and signature complexity criteria.

Brault and Plamondon proposed in [BP89] a measure of signature complexity, the “difficulty coefficient”, which is a function of the rate of geometric modifications such as length, direction of strokes and curvature per unit of time. This coefficient was used to accept or reject a signature at the enrolment step, by accepting a signature only if it is complex enough. The authors also proposed a “dissimilarity index” based on elastic matching between two signatures, for measuring the intra-class variability within the genuine signatures of a writer. They conclude that signers with low intra-class variability have a low rate of false acceptance and propose to select as references those signatures showing a low dissimilarity index. Di Lecce et al. [Di99] proposed a method to select reference signatures based on the analysis of stability in handwritten dynamic signatures. They compute signature stability as a sum of local stability indices. Elastic matching techniques are used to compute the correlation between different signatures of a writer, and a subset of signatures with the highest correlation is selected as reference set [Di02]. More recently, Guest and Fairhurst [GF06] carried out a sample signature selection at the enrolment step based on the assessment of the “Coefficient of Variance” (COV) for each global feature across all samples for a particular subject. The triplet of signatures with lowest COV value, namely with lowest variance, are selected as references. All such works of the literature point out the impact of complexity and stability criteria on improving the performance of signature verification systems.

The aim of this work is to propose an original approach for selecting the reference signatures of a writer based on a new complexity measure. Such measure is constructed by exploiting a global description of signatures in the frequency domain as well as a global statistical modelling of each signature instance. The hypothesis of this work is that the proposed complexity measure is fine enough for reflecting the variations of a writer’s signature from one instance to the next. For this reason, in order to select reference signatures, we exploit the distribution of complexity values of all the available genuine signatures. Experimental results on the widely used MCYT-100 database validate our hypothesis: the complexity measure characterizes well each genuine signature and can thus be used successfully for building a criterion to select reference signatures.

The organization of the paper is the following: Section 2 presents the complexity measure of a signature instance, Section 3 describes the experimental setup and the analysis of results. Finally, we conclude on the scope of this study in Section 4.

## 2 The novel complexity measure for selecting reference signatures

### 2.1 Quantifying complexity on the raw description of a signature instance

Online handwritten signatures are acquired on a digitizer, and according to this sensor's properties, different time functions are available (pen coordinates, pen pressure, pen inclination through time) [HG14]. In this study, we consider a signature as a raw sequence of pen coordinates  $(x(t), y(t))$  since this description of signatures is common to digitizers, tablets and smartphones. If such sequence of points representing an online signature is considered as being the outcome of a random variable, the concept of entropy can be used for estimating the degree of disorder associated to this random variable. The entropy of this variable depends on its associated probability density function [CT06]. To this end, an accurate estimation of the probability density associated to each signature instance must be achieved. We exploit for this purpose a Gaussian Mixture Model (GMM) [Re95], since this model has proven its efficiency in modeling signatures [MM08]. A GMM [Re95] is a weighted sum of  $M$  component Gaussian densities as given by the equation:

$$p(x|\lambda) = \sum_{i=1}^M w_i g(x|\mu_i, \Sigma_i) \quad (1.1)$$

where  $x$  is a  $D$ -dimensional continuous-valued data vector (i.e. feature vector),  $w_i$  for  $i = 1, \dots, M$ , are the mixture weights, and  $g(x|\mu_i, \Sigma_i)$ ,  $i = 1, \dots, M$ , are the component Gaussian densities. Each component density is a  $D$ -variate Gaussian function of the form,

$$g(x|\mu_i, \Sigma_i) = \frac{1}{(2\pi)^{D/2} |\Sigma_i|^{1/2}} \exp \left\{ -\frac{1}{2} (x - \mu_i)' \Sigma_i^{-1} (x - \mu_i) \right\} \quad (1.2)$$

with mean vector  $\mu_i$  and covariance matrix  $\Sigma_i$ .

The statistical complexity measure here proposed is based on the concept of differential entropy of information theory. For a given random variable  $X$  with a probability distribution  $f$ , the differential entropy  $h(x)$  is defined as follows:

$$h(x) = -\int_x f(x) \ln f(x) dx \quad (1.3)$$

For the multidimensional Gaussian distribution defined in Equation 1.2, such entropy has the following simplified form:

$$H(t) = \frac{1}{2} \ln \{ (2\pi e)^N \det(\Sigma) \} \quad (1.4)$$

For each signature of a given writer, we compute its complexity index as follows: the Gaussian component that gives the highest probability (the maximum value of the expression in Equation 1.2) is assigned to each point  $(x(t), y(t))$ . Then, we assign to the current point its corresponding differential entropy using Equation 1.4. For a signature sample of length  $N$ , the complexity index is defined as follows:

$$C = \frac{\sum_{t=1}^N H(t)}{N} \quad (1.5)$$

## 2.2 Quantifying complexity on the frequency domain of a signature instance

Fourier descriptors of a signature have already been used in the literature [KY08]. Fourier transform gives a global description of what happens in the temporal domain, by breaking down the signal into constituent sinusoids of different frequencies. The Fourier Transform coefficients of a given signal  $y(t)$  of length  $N$  are defined as follows:

$$C_k = \frac{1}{N} \sum_{t=0}^{N-1} y(t) e^{-j2\pi tk/N} \quad k=0,1,\dots,N-1. \quad (1.6)$$

For a given signature, Fourier analysis is carried out separately on  $x(t)$  and  $y(t)$ ,  $N$  being the number of points in the signature, and  $C_k$  the  $k$ -th Fourier coefficient  $C_k = a_k + jb_k$ . We exploit the magnitude of such coefficient, namely  $\sqrt{a_k^2 + b_k^2}$ , which measures the energy of the signal for the  $k$ -th harmonic. The resulting energy spectrum on  $x$  and  $y$  is then given as input to a GMM, this way using the same approach described in the previous section (2.1). Indeed, we aim at comparing a *global* description of signatures in the frequency domain, with its raw description in the time domain.

The next section presents our proposal of exploiting this complexity index computed on a signature instance for selecting the reference signatures of a given writer.

## 2.3 Selection of reference signatures based on the complexity index

Based on the complexity index above defined, we perform a Hierarchical Clustering in order to study the behavior of such measure on *all* genuine signature samples available.

Our study is carried out on the freely available and the widely used MCYT-100 subset of 100 persons [Or03]. We chose this database because it contains Western signatures of different styles, varying from simple flourish signatures to very complex flourish ones (rather close to cursive handwriting). Indeed, this allows assessing whether the complexity measure quantifies the existing gaps in complexity between different writers.

We determined the optimal number of clusters by computing different validity indices of the literature, namely Krzanowski-Laï index [DF02], Davies-Bouldin index [DB79], silhouette [Ro87], and Weighted intrer-intra index [St02]. The optimal number of clusters is 3, namely 3 categories of signatures according to their complexity, respectively displayed in Figure 1(a), 1(b) and 1(c).

The same validity indices are used to assess the optimal number of Gaussian components for the statistical model (GMM). We obtained that 24 mixture components is the optimal configuration because it optimizes the 4 validity indices, ensuring the best clustering.

In the same way, we assess the quality of the clustering on both the raw description of signatures and on their global description in the frequency domain. These 4 indices point out that the clustering on complexity values obtained after performing Fourier analysis on signatures is by far better than that obtained with the raw description of signatures.

### 2.3.1 Signatures' categories obtained by Hierarchical Clustering

As mentioned in the previous section, we retrieve 3 categories of signatures on the 2500 genuine signatures available. Figure 1 shows that each of such categories has a different degree of complexity and Table 1 gives the average complexity and its standard deviation per category. We clearly obtain a low complexity category (Figure 1(a)), a medium one (Figure 1(b)) and a high complexity category (Figure 1(c)). This result shows that our complexity index behaves well. Moreover, Figure 1(d) displays the values of the complexity index for all signatures per category, revealing that *its variance differs significantly between categories*. Indeed, this variance lowers with complexity; this can be seen in the upper part of Figure 1(d) (category in red of lowest variance), then in the medium complexity category with a higher variance (see complexity values in blue), and finally in the lowest complexity category with the highest variance (see values in green). This result confirms that complexity and stability are correlated in signatures as previously shown in the literature [BP89, GHD09].

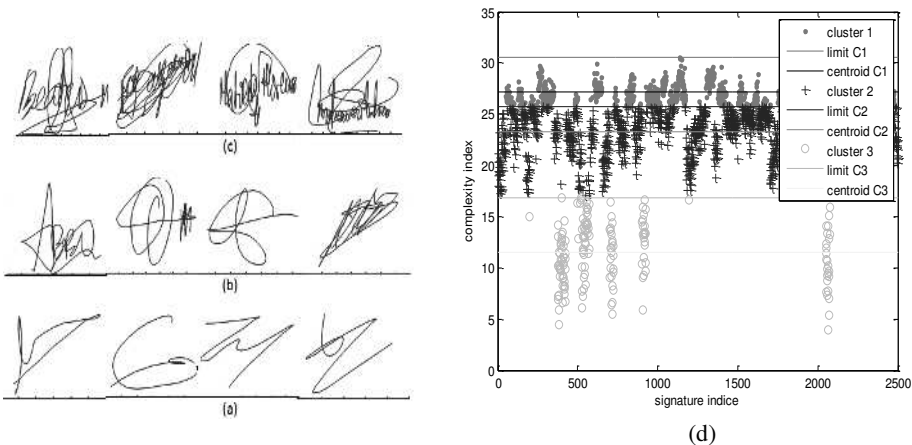


Figure 1: Examples of signatures in 3 complexity categories obtained by Hierarchical Clustering, (a) low, (b) medium, (c) high complexity. Such signatures were already published [Or03].

Complexity index-based categories	Percentage of signatures	Mean value	Std value
Low complexity	6.96%	11.50	2.9186
Medium complexity	51.16%	23.27	1.9494
High complexity	41.88%	27.15	0.9618

Table 1: Distribution of signatures of the MCYT-100 database in each complexity-based category; mean and standard deviation (Std) values of complexity per category.

### 2.3.2 The proposed method for selecting reference signatures

To select the best reference signatures, we analyze the distribution of complexity values on *all* genuine signature instances of a writer. The five nearest signatures to the median (indicated in red inside the boxplot of Figure (2b)) that is found between the first quartile ( $Q1=25\%$  of values) and the third quartile ( $Q3=75\%$  of values) are selected. Figure 2(a)

illustrates this method on the 25 genuine signatures from the first person in MCYT-100. This person belongs to the medium complexity category; note that complexity values of his/her signatures are spread in a quite large interval (17 to 24). This shows that the *intra*class variation is well reflected by our novel complexity measure since it is *sensitive to differences in signature instances of a same writer*. This fact has also an impact on the standard deviation of complexity values reported in Table 1.

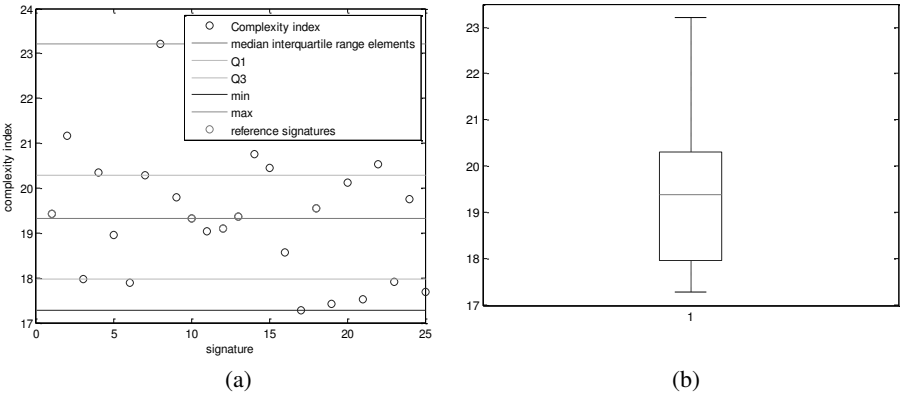


Figure 2: (a) The statistical distribution of the complexity index for all signatures and (b) the boxplot of the first person in MCYT-100 database.

### 3. Experiments

In the following, we evaluate the impact of the proposed method for selecting reference signatures in performance of a signature verification system. We compare the proposed method to a random selection of reference signatures. The 25 genuine signatures available per writer are used. The signature verification approach exploited for this evaluation is Dynamic Time Warping (DTW), proven to be one of the best approaches for signature verification [Ye04]. Concerning our method, reference signatures are selected in two ways: the 5 nearest to the median between quartiles Q1 and Q3 as explained above, and the 5 nearest to the mean value in the same interval (boxplot).

The random selection consists in sampling 5 reference signatures among the 25 genuine signatures of a given writer and consider his/her remaining 20 genuine signatures and the available 25 forgeries for verification purposes. We repeat the process 5 times. We compare in Figure 3 classifier performance with the 3 methods for reference signatures' selection: the 5 nearest to the median between quartiles Q1 and Q3, the 5 nearest to the mean value in the same interval (boxplot), and the random selection. Results are also displayed in Table 2 in terms of the Minimum Half Total Error Rate (minHTER). Our method for selecting signatures results in a significant relative improvement of 18% compared to the random selection. This result points out the pertinence of our approach that is based on an accurate complexity measure.

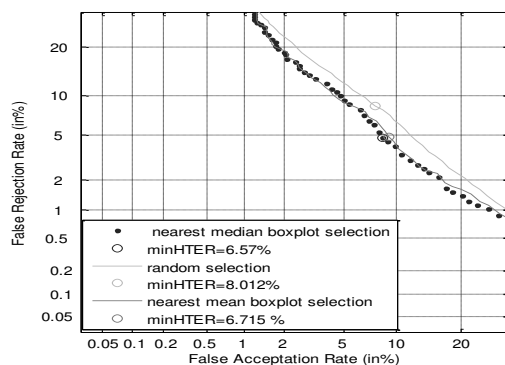


Figure 3: Detection Error Trade-off (DET)-Curves for selection of reference signatures based on complexity index.

Selection method	minHTER	Relative improvement compared to random selection
5 nearest to the median between Q1 & Q3	6.57%	18%
5 nearest to the mean between Q1 & Q3	6.715%	16%
5 random selected	8.012 %	—

Table 2: minHTER and relative improvement compared to random selection

## 4. Conclusions and future work

In this paper, a novel method for selecting reference signatures of a given writer is proposed. It is based on an original complexity measure that exploits a statistical global approach and a global description of signatures in the frequency domain. Experimental results reveal the effectiveness of the method, by generating a significant relative improvement of verification performance compared to a random selection of reference signatures. This proves that our complexity measure is not only able to reflect the gap in complexity between different writers and categories of writers, but even able to reflect *the variations in signature instances of a same writer*. In other words, it is *sensitive to intraclass variation* and thus an accurate tool for selecting references. Future work will be focused on studying how dynamic parameters have an influence on complexity, aiming at improving our selection method.

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