Optimum function for minimum weight evaluation of structural joints

L. T. Beilstein[1], S. Rudolph

Institute for Statics and Dynamics of Aerospace Structures
University of Stuttgart
Pfaffenwaldring 31
70569 Stuttgart

laura.beilstein@gmx.de
rudolph@isd.uni-stuttgart.de

Abstract: In this paper a general methodology for minimum weight selection of structural joints is reviewed. The evaluation and selection technique is based on analytical weight formulae for structural joints including different joining technologies for which through the use of Buckingham's Pi-Theorem a complete set of dimensionless similarity parameters is derived. This formal approach offers the possibility to evaluate and select the chosen design parameters in a scale-free form and ensures an optimal selection in terms of evaluation. The evaluation and selection approach represents a generic methodology for optimal choices in future knowledge-based design systems involving rule-based techniques.

1 Introduction

Accurate weight estimation in the early conceptual design stage in the aircraft industry is crucial since the performance characteristics of a design in subsequent design phases are highly affected by the weight. The ability to precisely predict weight in the early design phase will help to reduce costs and minimize design iterations.

Existing well established weight estimation formulae for structural components are often based on statistical methods [1, 2, 3]. The accuracy is, however, confined to such aircraft designs which are similar to designs of the underlying database. This apparent deficit clearly identifies the necessity for a physics-based augmentation. Hence, modern design approaches use both analytical and numerical calculations for weight prediction [4].

In the past, several strategies have been used either to optimize a structural concept, or to design a structure with regard to several design objectives (e. g. stiffness, strength, stability). The methodology of Ashby [5] for example focuses on material and process selection in mechanical design. The goal of the methodology reviewed in this paper is to provide a simple, fast and reasonably accurate weight forecast and evaluation method of structural joints at the conceptual design phase. Based on analytical weight formulae, a systematic approach is applied by using dimensional analysis in order to provide a generic evaluation method for minimum weight selection of different joining technologies.

---

[1] Formerly researcher at the Institute of Aircraft Design, University of Stuttgart

2 Analytical Weight Equations

Mass functions in symbolic form are derived from structural models (single lap joints under tensile load) and from analytical approaches for different joining technologies (e. g. riveting, adhesive bonding, welding) appropriate for conceptual design to optimize performance; which in this case is similar to weight minimization [6].

2.1 Riveted Joints

In the symbolic equation (1) a riveted single lap joint is exemplarily chosen and the dimensioning is based on performance of an ideal ductile material. Loads are transmitted through bearing stress [6]. The equation for minimum weight is as follows:

$$\frac{G_{\min}}{gbl^2} = \frac{j}{2C_{LA} R_m R_{m_N}^2 \, m \, \eta^2} \Big(C_{LD}\, j \big(4C_{LA} C_R + 2C_{LA}^2 (n-1) - n\pi\big) R_{m_N} \rho + C_{LD}\, jn \big(\pi R_{m_N} + 2\sqrt{3} R_m C_L \beta\big) \rho_N \Big) K^2 + \frac{j\rho}{R_m \eta} K \tag{1}$$

with $K = F/bl$ (structural index) [7].

3 Dimensional Analysis

The application of dimensional analysis to the relevance list of an equation results in a complete set of dimensionless products. These establish the groundwork for a systematic evaluation method suitable for weight comparison of different joining technologies.
For arbitrary complete physical equations $f(x_1,\dots, x_n) = 0$ of n dimensioned variables the Pi-Theorem is applicable. The modern formulation of Pi-Theorem according to Görtler [8] is: "From the existence of a complete and dimensionally homogeneous functional relationship $f(x_1, \dots, x_n) = 0$ of n physical quantities $x_i \in R^+$, the existence of a functional relationship $F(\pi_1,\dots, \pi_m) = 0$ with only $m = n - r$ dimensionless parameters π_j is always guaranteed". The dimensionless parameters π_j (also called similarity variables or dimensionless products) take the form

$$\pi_j = x_{r+j} \prod_{i=0}^{r} x_i^{-\alpha_{ji}} \tag{2}$$

where $j = 1,\dots,m \in N$ and $\alpha_{j1},\dots, \alpha_{jr} \in R$ and with r as the rank of the dimensional matrix formed by means of the n quantities, see Rudolph [9, 10] for more details.

3.1 Characteristic number for structural joints

The intensive examination with derived pi-sets from (2) leads to a definition of a significant, "characteristic" dimensionless number. The **weight efficiency factor π_{Weight}** is definied as shown in equation (3). With the dimensionless parameter $\pi_A=1/\pi_{Weight}$ only the additional weight due to the extra volume of the joint is considered which results in the **optimum function** (refer to equation (4)).

$$\pi_{Weight} = \frac{F}{G}\frac{l}{L_R} \tag{3}$$

with $L_R = R_m / g\,\rho$ (tension length)

$$\pi_{\Delta Weight} = \pi_{A_{Joint}} - \pi_{A_{Reference}} \tag{4}$$

with $\pi_A = 1/\pi_{Weight} = (G\,L_R)/(F\,l)$

3.2 Riveted Joints

From the relevance list of equation (1) the dimensional matrix can be build up. By rank preserving operations (i. e. elimination of redundancy[2]) the matrix is brought into a diagonal form [9]. From the dimensional matrix consisting of $n = 15$ physical quantities and a rank with $r = 3$, $m = n - r = 12$ dimensionless products are obtained. The defined dimensionless characteristic numbers ("pi-set") for the description as an optimum function have been introduced and the result is shown in Figure 1.

$$\pi_A = \frac{G}{F}\frac{L_R}{l} \qquad \pi_B = \frac{F}{R_m l b} \qquad \pi_C = n \tag{5}$$

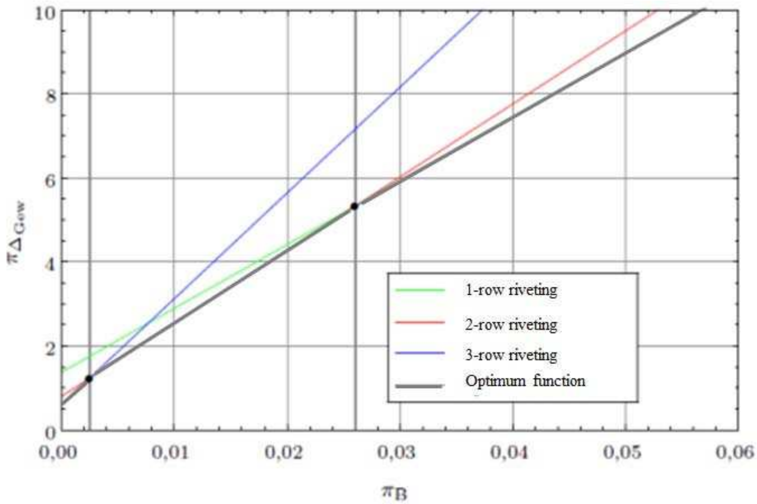

Figure 1 – Dimensionless additional weight of a single lap joint for riveted joints (1-row, 2-row, 3-row riveting) and the optimum function [6]

---

[2] redundancy-free representation is called "minimal" when the number of independent parameters cannot be further reduced [9].

Detailed analytical mass equations for structural joints (exemplarily selected) have been developed and, building on these, dimensionless parameters resulted from dimensionless analysis, make it possible to evaluate structural joints for minimum weight. For different joining technologies the dimensional analysis results in only three dimensionless products (refer to equation (5)). This pi-set enables a weight comparison by a so-called dimensionless optimum function [6] which can be used as a general evaluation method.

5 Conclusions

For aircraft manufacturers accurate weight prediction at early stages of preliminary design is vitally important to guarantee performance characteristics to customers. Current weight approximation formulae are often based on mathematical statistics which have been established in preliminary aircraft design due to their advantage of fewer input parameters and due to the lack of detailed CAD data. These known weight approximations result from data of existent aircraft and reflect the technology of the aircraft within that database. Therefore, alternative new approaches increasingly apply both analytical and numerical analyses. Besides, structural joints must be included in weight estimation processes to guarantee a more detailed and precise prediction result.
This paper presents and reviews a logical and scientifically stringent development of an evaluation strategy. The novel methodology has been used to determine analytical mass equations and to generate selection charts for different structural joints.
This concept of scale-free design knowledge enables an effective selection process for weight-minimal solutions for structural joints. Furthermore, this methodology represents a basic requirement for rule-based engineering and can be employed in implementing future applications of computer-aided aircraft design.

References

[1] D. P. Raymer, Aircraft Design: A Conceptual Approach, AIAA, Washington, D. C., 1989.

[2] J. Roskam, Airplane Design : Preliminary Sizing of Airplanes, Vol. I, DAR Corporation, Kansas, 2003.

[3] E. Torenbeek, Synthesis of Subsonic Airplane Design: an introduction to the preliminary design of subsonic general aviation and transport aircraft, with emphasis on layout, aerodynamic design, propulsion and performance, Kluwer Academic, Dordrecht, 1999.

[4] F. Hürlimann, Mass Estimation of Transport Aircraft Wingbox Structures with a CAD/CAE-Based Multidisciplinary Process, Dissertation, Eidgenössische Technische Hochschule, Zürich (2010).

[5] M. F. Ashby, Materials Selection in Mechanical Design, Elsevier, Amsterdam, 2005.

[6] L. Beilstein, Eine Methodik zur analytischen Gewichtsabschätzung und Bewertung von Strukturverbindungen im Flugzeugvorentwurf, Dissertation, Universität Stuttgart, Stuttgart (2012).

[7] J. Wiedemann, Leichtbau : Elemente und Konstruktion, Springer, Berlin, 2007.

[8] H. Görtler, Dimensionsanalyse: Theorie der physikalischen Dimensionen mit Anwendungen, Springer, Heidelberg, 1975.

[9] S. Rudolph, Übertragung von Ähnlichkeitsbegriffen, Habilitation, Universität Stuttgart, Stuttgart (2002).
[10] S. Rudolph, Eine Methodik zur systematischen Bewertung von Konstruktionen, Dissertation, Universität Stuttgart, Stuttgart (1994).