A tripartite architecture based on referee function for generic implementations of evidence combination rules

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Abstract: An approach for a generic implementation of combination rules of evidence is proposed. This approach implies a tripartite architecture, with respective parts implementing the logical framework (complete distributive lattice, Boolean algebra), the combination definition (referee function), and the belief-related processes (basic belief assignment, belief and plausibility computation, computations of the combinations according to their definitions). Referee functions are decisional arbitrament conditionally to basic decisions provided by the sources of information. Two generic processes are proposed for computing combination rule defioned by referee functions: a sampling method and a deterministic method based on an adaptive reduction of the set of focal elements. The proposed generic implementation makes possible the construction of several rules simply on the basis of a referee function extension.

Notations

- $I[boolean]$ is defined by $I[\mathbf{true}] = 1$ and $I[\mathbf{false}] = 0$.
- G^Θ denotes a *complete distributive lattice* or *Boolean algebra* generated by Θ ,
- $x_{1:n}$ and $\{x_j\}_{j=1:n}$ are abbreviations for the sequence $x_1, \cdots, x_n$.

1 Introduction

Evidence theory [1, 2] is often promoted as an alternative approach for fusing informations, when the hypotheses for a Bayesian approach cannot be precisely stated. When manipulating evidence-based informations, two issues may be encountered. First, there are potential combinatorics arising from the multiplication of the focal elements implied by the combination processes. Secondly, the interpretation of the evidence combination rules may be difficult, when conflicts are notably involved. In the recent litterature, there has been a large amount of work devoted to the definition of new fusion rules [3, 4, 6], in order to handle the conflict efficiently. The choice for a rule is often dependent of the applications and there is not a systematic approach for this task. Somehow, it appears also that this choice of a rule implies the choice of decision paradigm in order to handle the conflict. Facing such variety of rule, it appears that there is a need for tools as well as common frameworks, in order to evaluate and compare these various rules in regards to the applications.

This paper focuses on the definition of a system for the generic implementation of the combination rules. It allows an implementation of rules by means of simple and interpretable extensions of the system. All these extensions share common combination engines, which compute relaxation of the combinations and thus handle the combinatorics efficiently. Three keywords are thus the guideline of this work, *i.e* generic, efficient and interpretable.

Our approach is based on a tripartite architecture, with respective parts implementing the logical framework *(logical component)*, the combination definition *(referee component)*, and the belief-related processes *(belief component)*.

The logical framework represents the information while considered without its uncertainty. Typical logical frameworks are powersets, which are Boolean algebras. However, lattices are also useful frameworks and actually generalize the Boolean algebras. A common and incremental implementation is made by the logical component, adressing yet several frameworks.

The combination definition is obtained by implementing *referee functions*. Referee functions are decisional arbitraments conditionally to basic decisions provided by the sources of information. It is shown that referee functions [7] are sufficient for a definition of most combination rules. Then, a generic implementation of the rule is made possible on the basis of an algorithmic extension implementing the referee function.

The belief-related processes involve the encoding of basic belief assignment (bba), the computation of belief and plausibility from bba, but also the computations of the combinations according to their definitions. Two processes are proposed for computing the combination rules, both based on a definition of the combinations by means of referee functions: a sampling method and a summarization method, which provides an adaptive reduction of the set of focal elements. The purpose of these approaches is to avoid the combinatorics which are inherent to the definition of fusion rules of evidences.

Section 2 defines the logical component of our architecture. Section 3 defines the referee component. Section 4 introduces the combination processes of the belief component. Section 5 concludes.

2 Logical component

The logical component constitutes one of the three part of our system. While the combination of Dempster-Shafer [1, 2] is generally defined over powersets, the combination rules are defined on the basis of logical operators and properties which are not necessary inherent to powersets, nor Boolean algebras. Extensions of the theory have been proposed on the basis of distributive lattices [9]. Hyperpowersets are also distributive lattices. Most varients of evidence theory are built on logical structures known as *complete* (in general finite) *distributive lattices*. The logical component handles these structures in a generic manner, especially considering the logical notions from the general viewpoint of lattice.

These notions are defined now.

It is given Θ, a set of atomic propositions, which constitutes the logical frame of our knowledge representation. The set Θ is thus called the *frame of discernment*.

Distributive lattice: An algebraic structure G^Θ generated by Θ and operators $\cap$ and $\cup$ is a distributive lattice if it verifies the properties:

- $X \cup (Y \cup Z) = (X \cup Y) \cup Z$; $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (associativity)
- $X \cup Y = Y \cup X$; $X \cap Y = Y \cap X$ (commutativity)
- $X \cup (X \cap Y) = X$; $X \cap (X \cup Y) = X$ (absorption)
- $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ (distributivity)

Implied order: The partial order $\subseteq$ is implied by $X \subseteq Y \overset{\Delta}{\Longleftrightarrow} X \cap Y = X \Longleftrightarrow X \cup Y = Y$.

Complete distributive lattice: A distributive lattice G^Θ is *complete* if any subset $\Xi \subset G^\Theta$ has a greatest lower bound and a least upper bounds. This property is implied by the finiteness of the structures in use.

Complement and co-complement: The complement, $\overline{X}$, and the co-complement, $\underline{X}$, are defined by:

$$\overline{X} = \bigcup_{Y:Y\cap X=\emptyset} Y \quad\text{and}\quad \underline{X} = \bigcap_{Y:Y\cup X=\Omega} Y\,.$$

Complete Boolean algebra: A complete distributive lattice G^Θ is a complete Boolean algebra if $\underline{X}\cap X = \emptyset$ and $\overline{X}\cup X = \Omega$ for any $X \in G^\Theta$. As a consequence, the complement and co-complement operators are the same for a complete Boolean Algebra.

Examples: A powerset, 2^Θ, generated by Θ is a complete Boolean algebra. An *hyper-power set*, D^Θ, generated by a *finite* Θ is a complete distributive lattice.

3 Referee component

The referee component constitutes the part of our system, which implements the definition of the combination rules. The definition of the combination rules is made by means of *referee function*. Referee functions have been defined in [7] and allow a simple, general and computational interpretation of the combination rules. This interpretation was inspired first by works on probabilistic restrictions of evidence combinations. This work led to a conditionnal interpretation of the combination, in terms of *Referee Function*.

3.1 Referee function

Definition A referee function over G^Θ for s sources of information and with context γ is a mapping $X, Y_{1:s} \mapsto F(X|Y_{1:s};\gamma)$ defined on propositions $X, Y_{1:s} \in G^\Theta$, which satisfies for any $X, Y_{1:s} \in G^\Theta$:

$$F(X|Y_{1:s};\gamma) \geq 0 \text{ and } \sum_{X \in G^\Theta} F(X|Y_{1:s};\gamma) = 1 \,,$$

A referee function for s sources of information is also called a s-ary referee function. The quantity $F(X|Y_{1:s};\gamma)$ is called a *conditional arbitrament* between $Y_{1:s}$ in favor of X. Notice that X is not necessary one of the propositions $Y_{1:s}$; typically, it could be a combination of them. The case $X = \emptyset$ is called the *rejection case*.

Fusion rule Let be given s basic belief assignments (bba) $m_{1:s}$ and a s-ary referee function F with context $m_{1:s}$. Then, the fused bba $m_1 \oplus \cdots \oplus m_s[F] \triangleq \oplus[m_{1:s}|F]$ based on the referee F is constructed as follows:

$$\begin{aligned} &\oplus[m_{1:s}|F](X) = \frac{I[X \neq \emptyset]}{1-z} \sum_{Y_{1:s} \in G^\Theta} F(X|Y_{1:s};m_{1:s}) \prod_{i=1}^{s} m_i(Y_i) \,, \\ &\text{where } z = \sum_{Y_{1:s} \in G^\Theta} F(\emptyset|Y_{1:s};m_{1:s}) \prod_{i=1}^{s} m_i(Y_i) \,. \end{aligned} \tag{1}$$

The value z is called the *rejection rate*.

3.2 Examples of referee functions

Dempster-shafer: The definition of a referee function for Dempster-Shafer combination is immediate:

$$m_{\mathrm{DS}} = \oplus[m_{1:s}|F_{\mathrm{DS}}] \,, \text{ where } F_{\mathrm{DS}}(X|Y_{1:s};m_{1:s}) = I\left[X = \bigcap_{k=1}^{s} Y_k\right] \,.$$

Disjunctive: The definition of a referee function for the disjunctive combination is:

$$m_{\mathrm{d}} = \oplus[m_{1:s}|F_{\mathrm{d}}] \,, \text{ where } F_{\mathrm{d}}(X|Y_{1:s};m_{1:s}) = I\left[X = \bigcup_{k=1}^{s} Y_k\right] \,.$$

PCR6: The proportional conflic redistribution rules (PCRn) have been introduced By Dezert and Smarandache [6] and the rule PCR6 has been proposed by Martin and Osswald in [5] . The original definition of the rule could be found there. A formulation of PCR6 by means of a referee function is derived in [7]:

$$m_{\mathrm{PCR6}} = \oplus[m_{1:s}|F_{\mathrm{PCR6}}] \ ,$$

where the referee function F_{PCR6} is defined by:

$$F_{\mathrm{PCR6}}(X|Y_{1:s};m_{1:s}) = I\left[X = \bigcap_{k=1}^{s} Y_k \neq \emptyset\right] + I\left[\bigcap_{k=1}^{s} Y_k = \emptyset\right] \frac{\sum_{j=1}^{s} I[X = Y_j]\ m_j(Y_j)}{\sum_{j=1}^{s} m_j(Y_j)} \ . \tag{2}$$

This referee function implies an interpretation of PCR6 as a two-cases process:

- The entry informations are compatible; then, the conjunctive consensus is decided,
- The entry informations are not compatible; then, a mean decision is decided, weighted by the relative beliefs of the entries.

PCR6 only considers consensus or no-consensus cases. But for more than 2 sources, there are many cases of *intermediate consensus*. By construction, PCR6 is not capable to manage intermediate consensus. This limitation is addressed by rule PCR# introduced in [7] and which is defined in the long version of the paper.

4 Belief component: combination

Two combination processes are defined, which are by sampling and by summarization. Both processes work on a same definition of the combination based on referee functions.

Process based on sampling. The process produces an approximation of the fused bba by means of a cloud of particles, which are elements of G^{Θ}.

The algorithm is described in the long version of the paper.

Process based on summarization. The principle is to limit the size of the set of focal elements by reducing this size by a simple summarization process[8]. In our approach, the less significant focal elements in term of basic belief assignment are both relaxed to their union, and the process is repeated as many times it is needed.

The algorithm is described in the long version of the paper.

5 Conclusion

This paper describes an architecture for generic implementation of combination rules of evidence. This architecture is tripartite, that is it is composed of a logical part, a definition part and a processing part. The logical part specifies on which kind of logical framework the rule is defined. The definition part specifies the combination itself, and is implemented by means of referee functions, which are arbitrament processes conditionally to the contributions of the sources of information. Altogether, these components make the architecture easily expandable and interpretable. This architecture is actually implemented under the form of a java package, and is made available at address:

`http://refereefunction.fredericdambreville.com/releases`

Future releases will focus on performance improvement and on the implementation of more combinations rules.

Literatur

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