Performance analysis of passive emitter tracking using TDOA, AOA and FDOA measurements

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Abstract: This paper investigates passive emitter tracking using a combination of Time Difference of Arrival (TDOA) measurements with further different types of measurements. The measurements are gained by exploiting the signal impinging from an unknown moving emitter. First, a combined set of TDOA and Angle of Arrival (AOA) measurements is processed using the Maximum Likelihood Estimator (MLE). Then, a Gaussian Mixture (GM) filter is used to solve the tracking problem based on TDOA and Frequency Difference of Arrival (FDOA) measurements. In Monte Carlo simulations, the superior performance of the combined methods in contrast to the single TDOA approach is shown and compared with the Cramér-Rao Lower Bound (CRLB).

1 Introduction

Many applications require fast and accurate localization and tracking of non-cooperative emitters. In many cases, it is advantageous not to conceal the observation process by using active sensors, but to work covertly with passive sensors. The estimation of the emitter state is based on various types of passive measurements obtained by exploiting signals emitted by targets, [KMK10, Bec05, and references therein].

Measurements of Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) are obtained from a network of several spatially dislocated sensors. Here, a minimum of two sensors is often needed.

In the absence of noise and interference, a single TDOA measurement localizes the emitter on a hyperboloid with the two sensors as foci. By taking additional independent TDOA measurements from at least four sensors, the three-dimensional emitter location is estimated from the intersections of three or more hyperboloids. If sensors and emitter lie in the same plane, one TDOA measurement defines a hyperbola of possible emitter locations. That is why, the localization using TDOA measurements is called hyperbolae positioning. The sign of the measurement defines the branch of the hyperbola on which the emitter is located. The two-dimensional emitter location is found at the intersection of two or more hyperbolas from at least three sensors. This intersection point can be calculated by an analytical solution, see e.g. [SHH08, HY08].

Alternatively, a pair of two sensors moving along arbitrary but known trajectories can be used for localizing an emitter using TDOA measurements. In this case, the emitter location can be estimated by filtering and tracking methods based on further measurements over

time. The localization of an unknown, non-cooperative stationary emitter using TDOA measurements from a sensor pair has already been investigated in [Kau09].
In this paper, the gain in performance by combining TDOA measurements from one sensor pair with further different kinds of passive measurements is analyzed . The focus is on localization and tracking a non-cooperative moving emitter.
Firstly, TDOA measurements are combined using additional Angle of Arrival (AOA) measurements from one of the two sensors. The measurement set based on the TDOA and the combination of TDOA and AOA measurements are processed using the Maximum Likelihood Estimator (MLE). Secondly, measurement information is increased by FDOA measurements which are gained by differentiating the Frequencies of Arrival (FOA) of the sensor pair. In this case, relative motion between sensors and emitter is necessary. The Gaussian Mixture (GM) filter described in [Kau09] is used to obtain comparable results of the single TDOA and the combined TDOA/ FDOA method. In Monte Carlo simulations, the performance of the different methods is analyzed and compared with the Cramér-Rao Lower Bound (CRLB).

2 Problem description

A two-dimensional emitter-sensors scenario is considered. Let $\mathbf{e}_k = (\mathbf{x}_k^T, \dot{\mathbf{x}}_k^T)^T$ be the emitter state at time t_k, where $\mathbf{x}_k = (x_k,\ y_k)^T \in \mathbb{R}^2$ denotes the position and $\dot{\mathbf{x}}_k = (\dot{x}_k,\ \dot{y}_k)^T \in \mathbb{R}^2$ the velocity. Two sensors with the state vectors

$$\mathbf{s}_k^{(i)} = \left(\mathbf{x}_k^{(i)T}, \dot{\mathbf{x}}_k^{(i)T}\right)^T, \qquad i = 1, 2, \tag{1}$$

observe the emitter and receive the emitted signal. They move along arbitrary but known trajectories.
The emitter is assumed to move with constant velocity. Therefore, the emitter state can be modeled from the previous time step t_{k-1} by adding white Gaussian noise:

$$\mathbf{e}_k = \mathbf{F}_k\, \mathbf{e}_{k-1} + \mathbf{v}_k, \quad \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}), \tag{2}$$

with

$$\mathbf{F}_k = \begin{pmatrix} 1 & 0 & t_k - t_{k-1} & 0 \\ 0 & 1 & 0 & t_k - t_{k-1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

where $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ means that $\mathbf{v}_k$ is zero-mean normally distributed with covariance $\mathbf{Q}$.
The TDOA measurement in the range domain is given by:

$$h^{\mathrm{r}}(\mathbf{e}_k) = ||\mathbf{x}_k - \mathbf{x}_k^{(1)}|| - ||\mathbf{x}_k - \mathbf{x}_k^{(2)}||, \tag{4}$$

where $||\cdot||$ denotes the vector norm and $\mathbf{r}_k^{(i)} = \mathbf{x}_k - \mathbf{x}_k^{(i)}$ denotes the relative position vector between the emitter and sensor i. The measurement process is modeled by adding white Gaussian noise to the measurement function:

$$z_k^{\mathrm{r}} = h^{\mathrm{r}}(\mathbf{e}_k) + u_k^{\mathrm{r}}, \quad u_k^{\mathrm{r}} \sim \mathcal{N}(0, \sigma_{\mathrm{r}}^2) \tag{5}$$

where σ_{r} denotes the standard deviation of the measurement error in the range domain. The measurement noise u_k^{r} is i.i.d., the measurement error is independent from time to time, i.e. mutually independent, and identically distributed.

3 Combination of TDOA and AOA measurements

Let $\mathbf{s}^{(1)}$ be the location of the sensor, which takes the bearing measurements. The additional azimuth measurement function at time t_k is:

$$h^{\alpha}(\mathbf{e}_k) = \arctan \frac{x_k - x_k^{(1)}}{y_k - y_k^{(1)}} \tag{6}$$

Addition of white noise yields:

$$z_k^{\alpha} = h^{\alpha}(\mathbf{e}_k) + u_k^{\alpha}, \quad u_k^{\alpha} \sim \mathcal{N}(0, \sigma_{\alpha}^2), \quad u_k^{\alpha} \text{ i.i.d.}, \tag{7}$$

where σ_{α} is the standard deviation of the AOA measurement. AOA and TDOA measurement noise are assumed to be uncorrelated from each other.
At each time step, one azimuth and one TDOA measurement are taken. The azimuth measurement defines a line of possible emitter locations and the TDOA measurement localizes the emitter on a hyperbola. This pair of nonlinear measurements must be processed with nonlinear estimation algorithms.

CRLB

It is important to know the optimal estimation accuracy that can be achieved with the available measurements for the problem under consideration. The CRLB, the inverse of the Fisher information $\mathbf{J}$, provides a lower bound on the estimation accuracy for an unbiased estimator. The CRLB of the combined measurement set is calculated over the fused Fisher information. The Fisher information matrix (FIM) at time t_k is the sum of the FIMs based on the TDOA and the AOA measurements:

$$\mathbf{J}_k = \frac{1}{\sigma_{\mathrm{r}}^2} \sum_{i=1}^{k} \left(\frac{\partial h^{\mathrm{r}}(\mathbf{e}_i)}{\partial \mathbf{e}_k} \right)^T \frac{\partial h^{\mathrm{r}}(\mathbf{e}_i)}{\partial \mathbf{e}_k} + \frac{1}{\sigma_{\alpha}^2} \sum_{i=1}^{k} \left(\frac{\partial h^{\alpha}(\mathbf{e}_i)}{\partial \mathbf{e}_k} \right)^T \frac{\partial h^{\alpha}(\mathbf{e}_i)}{\partial \mathbf{e}_k}, \tag{8}$$

where

$$\frac{\partial h^{\mathrm{r}}(\mathbf{e}_i)}{\partial \mathbf{e}_k} = \frac{\partial h^{\mathrm{r}}(\mathbf{e}_i)}{\partial \mathbf{e}_i} \frac{\partial \mathbf{e}_i}{\partial \mathbf{e}_k}. \tag{9}$$

Therefore, the localization accuracy depends on the sensor-emitter geometry and the standard deviation of the TDOA and the azimuth measurements.

4 Combination of TDOA and FDOA measurements

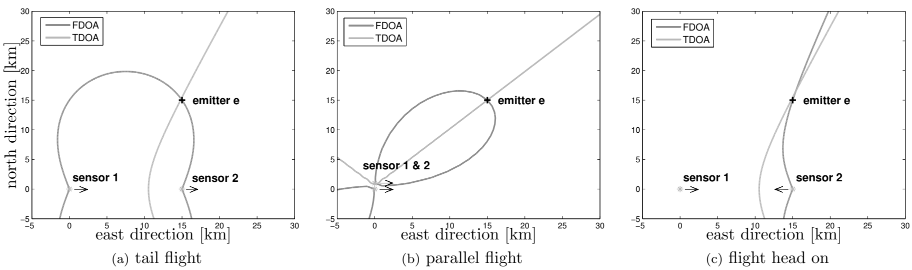

Figure 1: Combination of TDOA and FDOA measurements in three different scenarios

The FDOA measurement function depends not only on the emitter position but also on its speed and course: $h^{\mathrm{ff}}(\mathbf{e}_k) = \frac{f_0}{c} h^{\mathrm{f}}(\mathbf{e}_k)$, where f_0 is carrier frequency of the signal. Multiplication with $\frac{c}{f_0}$ yields the measurement equation in the velocity domain:

$$h^{\mathrm{f}}(\mathbf{e}_k) = (\dot{\mathbf{x}}_k^{(1)} - \dot{\mathbf{x}}_k)^T \frac{\mathbf{r}_k^{(1)}}{||\mathbf{r}_k^{(1)}||} - (\dot{\mathbf{x}}_k^{(2)} - \dot{\mathbf{x}}_k)^T \frac{\mathbf{r}_k^{(2)}}{||\mathbf{r}_k^{(1)}||}. \tag{10}$$

Under the assumption of uncorrelated measurement noise from time step to time step and from the TDOA measurements, we obtain the FDOA measurement equation in the velocity domain:

$$z_k^{\mathrm{f}} = h^{\mathrm{f}}(\mathbf{e}_k) + u_k^{\mathrm{f}}, \quad u_k^{\mathrm{f}} \sim \mathcal{N}(0, \sigma_{\mathrm{f}}^2), \tag{11}$$

where σ_{f} is the standard deviation of the FDOA measurement. The associated TDOA/ FDOA measurement pairs may be obtained by using the Complex Ambiguity Function ([St81]).

Fig. 1 shows the situation for different sensor headings after taking one pair of TDOA and FDOA measurements. The green curve, i.e. the branch of hyperbola, indicates the ambiguity after the TDOA measurement. The ambiguity after the FDOA measurement is plotted in magenta. The intersection of both curves presents a gain in information for the emitter location. This gain is very high if sensors perform a tail flight, see Fig. 1 (a).

CRLB

The Fisher information at time t_k is the sum of the Fisher information based on the TDOA and the FDOA measurements:

$$\mathbf{J}_k = \frac{1}{\sigma_{\mathrm{r}}^2} \sum_{i=1}^{k} \left(\frac{\partial h^{\mathrm{r}}(\mathbf{e}_i)}{\partial \mathbf{e}_k} \right)^T \frac{\partial h^{\mathrm{r}}(\mathbf{e}_i)}{\partial \mathbf{e}_k} + \frac{1}{\sigma_{\mathrm{f}}^2} \sum_{i=1}^{k} \left(\frac{\partial h^{\mathrm{f}}(\mathbf{e}_i)}{\partial \mathbf{e}_k} \right)^T \frac{\partial h^{\mathrm{f}}(\mathbf{e}_i)}{\partial \mathbf{e}_k}. \tag{12}$$

The FIM of the FDOA measurements not only depends on the geometry and the standard deviation of the measurements. But also they strongly depend on the relative speed vectors between emitter and sensors.

5 Simulation Results

A moving emitter is considered to compare the performance of the single TDOA approach and the combined methods. Fig. 2 (a) shows the measurement situation. Sensors fly one after another at a constant speed of 50 m/s and perform one maneuver to ensure observability. The emitter moves at a constant velocity of 40 m/s in east south direction. TDOA and FDOA measurements are gained from the network of the two sensors. Sensor $\mathbf{s}^{(1)}$ takes the azimuth measurements. TDOA measurement standard deviation in the range domain is assumed to be 100 m (0.33 μs), the standard deviation of the azimuth measurements is assumed to be 3 degree and FDOA measurement standard deviation is assumed to be 1 m/s (assuming a carrier frequency of 3 GHz this corresponds to 10 Hz in the frequency domain, the carrier frequency is assumed to be known).
The results shown here are the product of 1000 Monte Carlo runs with a sampling interval of two seconds. A total of 120 s is regarded.
Firstly, the combined measurement set of TDOA and AOA measurements is processed using the MLE which is implemented with the simplex method due to Nelder and Mead as numerical iterative search algorithm. For each Monte Carlo run, the emitter state is computed once processing the complete measurement set. In comparison, a MLE only based on TDOA measurements is performed. This MLE is initialized with a point generated randomly from a Gaussian distribution centered at the true emitter location with standard deviation of 500 m in the x and y direction. The velocity entries are set to 10. In contrast, the intersection point of the first TDOA/AOA measurement pair provides the initialization point of the combined method. In Fig. 2 (b), the results of the combination of TDOA and AOA are labeled with the acronym TAOA MLE, the results of the TDOA approach with the acronym TDOA MLE.

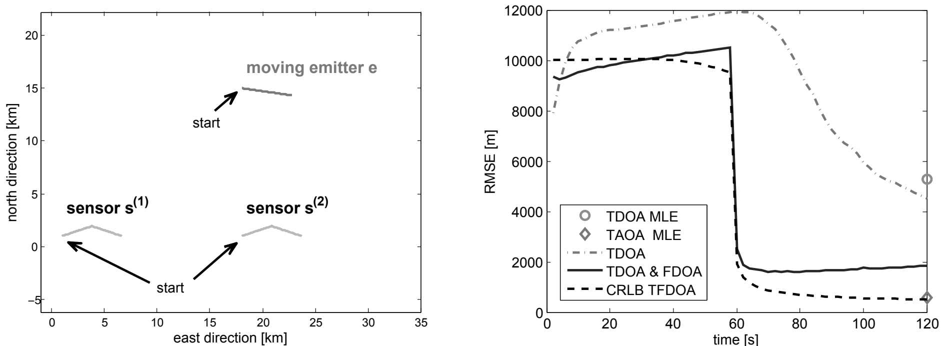

Figure 2: (a) scenario and (b) RMSE for TDOA, TDOA and AOA, and TDOA and FDOA

Secondly, the combination of TDOA and FDOA is exploited using the static GM filter described in [Kau09]. Initially, the first TDOA measurement is approximated by a GM in the Cartesian state space incorporating prior information about the area in which the emitter must lie. Additionally, a maximal emitter speed of 60 m/s in x and y direction is assumed. The updates are done using EKFs for the incoming TDOA and FDOA measurements. The performance of the combined filter (acronym TDOA & FDOA) is compared to the GM

filter only based on TDOA measurements (acronym TDOA).
In Fig. 2 (b), the Root Mean Square Errors (RMSE) of the different algorithms are plotted against the time in seconds. It shows the superior performance of the combined methods. Due to the initialization, the GM filter are in the initial phase better than the CRLB. CRLB for the TDOA/FDOA approach are shown with additional assumptions (CRLB TFDOA). In this scenario, the best results are obtained with the combination of TDOA and AOA.

6 Conclusions

For passive emitter tracking in sensor networks different measurement types can be obtained by exploiting the signal impinging from the target. Some of them can be taken by single sensors; e. g. bearing measurements. Others can only be collected by a network of sensors. A minimum of two sensors is needed here. TDOA and FDOA measurements belong to this group. FDOA measurements are highly dependent on the relative motion between emitter and sensors.
The combination of different measurements leads to a significant gain in estimation accuracy. In the investigated scenario, this gain is very high in the case of combining TDOA and AOA measurement information.

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