# **Analysis of Minutiae Quality for Improved Workload Reduction in Fingerprint Identification**

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**Abstract:** The workload of biometric identification in large fingerprint databases poses a challenging problem. Efficient schemes for biometric workload reduction are a topic of ongoing research. Some of the state-of-the art approaches rely on triangles of minutia points generated by Delaunay triangulation, which are then used for indexing. In this paper, we investigate how quality estimation at the minutia level can improve the performance of such algorithms and hence the system workload. In order to reduce the number of spurious and missing minutiae, we analyse the impact of selecting minutiae points based on their qualities. This, in turn, can significantly distort the triangulation. In addition, we consider the usefulness of the average minutia quality as an additional criteria of the minutia triangles for indexing. Our results show that both strategies lead to a significant reduction in biometric workload compared to a baseline solution (*i.e.* exhaustive search) – down to 36% on average.

**Keywords:** Computational workload-reduction, indexing, fingerprint identification, minutiae quality, Delaunay triangulation.

# 1 Introduction

Fingerprints are one of the most popular biometric characteristics deployed in many applications such as unlocking consumer smartphones, forensic investigations, and national ID systems. However, the rapid growth in the number of subjects enrolled in these systems (e.g. more than a billion in the Indian national ID system Aadhaar [UI12]) leads to a high workload and long transaction times in a biometric identification system.

Biometric identification is the process of searching a biometric enrolment database in order to find and return the biometric reference identifier(s) attributable to a single individual [IS21]. Evidently, this mode of operation requires the processing of large amounts of biometric data, as a biometric probe is usually compared with all stored biometric references (*i.e.* a one-to-many biometric comparison). Here, biometric identification systems depending on such exhaustive searches lead to a high system workload which is dominated by comparison costs. That is, the number of comparisons grows linearly with the number of enrolled subjects. In this context, biometric *workload reduction* (WR) methods [DRB19], *a.k.a.* biometric indexing schemes, have been proposed to reduce the overall computational effort (*e.g.* in terms of the number of comparisons) in biometric identification transactions.

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Fig. 1: Examples of fingerprint samples leading to different possible triangles according to different numbers of minutiae selected: 9, 12, and 15 minutiae points are selected per fingerprint sample. Fingerprint samples taken from the database FVC2006 [Ca07], subset DB2(optical sensor).

Recently, the utilities<sup>3</sup> of fingerprint samples have been turned into an advantage for rapid indexing and hence reducing the number of comparisons per biometric identification transaction [Os22]. More importantly, it has been shown that the character of a fingerprint sample (*e.g.* referring to attributes such as creases, scars, dermatological issues) is a stable contributor to sample quality [IS16], and thus, for indexing.

Fingerprint sample quality is an essential factor for WR solutions and many biometric deployments [GHB18]. Fingerprint samples from a single instance may differ in rotation, translation, scale (when captured by different sensors) and most notably in the minutiae points, that are detected. Hence, schemes that search for minutiae triplets are typically costly, *i.e.*  $O(n^3)$  where n is the number of minutia points detected [KK17]. It should be noted that only a subset of possible minutia combinations are considered, often derived by Delaunay triangulation [Ka14]. In that case, missing or spurious minutiae are a challenging problem as they can change the triangulation outcome significantly. Even, for Delaunay triangulation-based indexing schemes, the number of generated triangles can be low, non-stable, and with loss of geometric features. Fig. 1 depicts some examples of fingerprint samples varying their triangle outcomes.

In this paper, we investigate the effect of the estimation of minutiae quality into a well-known minutiae triplet computation approach called expanded Delaunay triangulation [Ka14], which in turn can be integrated in a computational WR scheme to reduce the number of required comparisons per identification transaction. To that end, simple geometric properties (e.g. largest angles, the lateral length of the longest side) are exploited to build triangles suitable for indexing fingerprint templates. In addition, we investigate how the average minutiae quality as a criteria to represent a triangle can contribute to the geometric properties already binned and hence drastically reduce the search space. The experimental results reported on well known databases varying the type of sensor, show that removing low quality minutiae prior to expanded Delaunay triangulation leads to a low penetration rates in a fingerprint biometric identification system.

<sup>&</sup>lt;sup>3</sup> Utility is a score computed by an biometric sample quality assessment algorithm.

Tab. 1: Overview of most relevant published minutiae triplet search indexing-based schemes that employ Delaunay triangulation in fingerprint-based identification systems (results are reported for best configurations and scenarios).

Approach	WR category	Quality information	Dataset	Type of sensor	Workload-reduction(%)
Ross and Mukherjee [RM07]	pre-selection	No	FVC2002	DB1(optical) DB2(optical) DB3(capacitive) DB4(synthetic)	HR=100,PR=47.32 HR=100,PR=47.07 HR=100,PR=50.27 HR=100,PR=45.39
			FVC2004	DB1(optical) DB2(optical) DB3(thermal Sweep) DB4(synthetic)	HR=100,PR=51.40 HR=100,PR=46.97 HR=100,PR=52.41 HR=100,PR=51.40
Khachai et al. [KLD14]	pre-selection	No	NIST DB4	-	CIP=82,PR=5
Liang et al. [LAB06]	pre-selection	No	FVC2000	DB2(capacitive)	CIP=100,PR=29
Uz et al. [Uz09]	feature transformation	Yes (minutiae level)	FVC2000	DB1(optical)	Avg.#=59.23 w.r.t. a total of minutiae
	pre-selection	Yes (minutiae level)	NIST DB4	-	CIP=98,PR=30
			FVC2006	DB2(optical)	CIP=99,PR=30
Gago-Alonso et al. [Ga13]			FVC2000	DB1(optical) DB2(capacitive) DB3(optical)	CIP=99,PR=30 CIP=99,PR=30 CIP=98,PR=30
			FVC2004	DB1(optical)	CIP=99,PR=30
Kavati <i>et al.</i> [Ka14]	pre-selection	No	FVC2004	DB1(optical) DB2(optical) DB4(synthetic)	HR=99,PR=60 HR=99,PR=41 HR=98,PR=58
			FVC2002	DB1(optical) DB2(optical) DB3(capacitive) DB4(synthetic)	HR=99,PR=16 HR=99,PR=15 HR=98,PR=40 HR=98,PR=40
Khodadoust and Khodadoust [KK17]	pre-selection	Yes (image level)	FVC2000	DB1(optical) DB2(capacitive) DB3(optical)	HR=98,PR=10 HR=95,PR=10 HR=93,PR=10
			FVC2004	DB1(optical)	HR=94,PR=10
			NIST DB4	-	HR=95,PR=10
			NIST DB14		HR=94,PR=10
Ours	pre-selection	Yes (minutiae level)	FVC2006	DB1(electric) DB2(optical) DB3(thermal) DB4(synthetic)	HR=100,PR=36.15 HR=100,PR=31.17 HR=100,PR=35.81 HR=100,PR=39.98

HR: Hit Rate, PR: Penetration Rate, CIP: Correct Index Power, Avg.#: Average number of minutiae points used in a one-to-one comparison.

The remainder of this paper is organised as follows: Sect. 2 briefly introduces the related work. In Sect. 3, the proposed system is described in detail. Sect. 4 presents the experimental setup and the achieved results, while a summary and concluding remarks are given in Sect. 5.

# 2 Related work

Different fingerprint indexing schemes based on minutiae triplet information have been designed to reduce workload in terms of the number of comparisons (*i.e.* pre-selection schemes-based) in a fingerprint identification system. Tab. 1 shows an overview of the minutiae triplet search indexing-based schemes. Most of these approaches take advantage of geometric properties that can be inferred from the minutiae information over triangu-

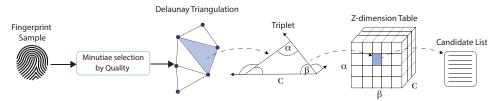


Fig. 2: Overview of the proposed indexing system utilizing minutiae quality.

lar properties. Ross et al. [RM07] proposed indexing minutiae triplets by including the ridges curve information as a feature defined by the Delaunay triangulation. Khachai et al. [KLD14] also suggested this type of triangulation as it allows maximising the minimum angle of each triangle computed, even, for distorted fingerprint samples [LAB06]. Further approaches have included the quality information as a criteria to improve the process of Delaunay triangulation [Uz09] for fingerprint comparisons. This enables a rapid comparison into a hierarchical Delaunay triangulation, where the top level only contains the high quality minutiae, and the bottom level are all the possible minutiae, independent of quality. In particular, this strategy is designed to a feature transformation-based WR. Thus, WR is limited to reduce the average minutiae number in a one-to-one comparison. On the other hand, Gago et al. [Ga13] proposed an expanded triangulation for indexing which discards the minutiae that are in the border of the impressions or in bad quality areas (false minutiae). In this context, bad quality areas are globally defined by the coherence and orientation of minutiae maps [REB10]. Despite this mechanism tries to solve the problem of missing and spurious minutiae, there are still erroneous features produced by fingerprints containing scars. More recently, in [KK17], elliptical properties for indexing are defined on an expanded Delaunay triangulation computed on different minutiae quality levels (i.e. high, medium, and low). However, it should be noted that the minutiae quality is defined through the sample quality and not from a minutiae extractor. Finally, thresholds to group minutiae based on their qualities and define discrete categories are less flexible for enrolment and indexing proposals across different sensors.

## 3 Proposed system

Fig. 2 shows a conceptual overview of the improved minutiae quality-based indexing and retrieval. The proposed approach employs well-known techniques and properties from the literature for fingerprint indexing. As mentioned in Sect. 1, the main contribution of this scheme is to show how the analysis of the minutiae quality can improve further minutiae triplet search process (*e.g.* expanded Delaunay-based triangulation [Ka14]) and thus the reduction of the number of comparisons per identification transaction. The proposed method follows similar steps as proposed in [Ka14]: *i*) computation of the expanded triangle set per fingerprint<sup>4</sup>; note that different triangles can be generated from different number of minutiae points, *ii*) building a bin taking into account some characteristics of the triangles

<sup>&</sup>lt;sup>4</sup> This includes the triangles in the Delaunay triangulation and the triangles in the triangular hulls of all the minutiae points contained in the sample [Ka14].

Tab. 2: Different types of triplets used on the proposed scheme. Note that different triangle characteristics are clustered and empirically computed in order to build different bin tables for indexing. It is important to know that such bin tables are used in an independent way in our scheme, not all at once.

Type of triplet	Bins
PROP1	$b_i  o (\alpha, \beta, c)$
PROP2	$b_i  ightarrow ( heta_1,  heta_2)$
PROP1-PROP2	$b_i \rightarrow (\alpha, \beta, c, \theta_1, \theta_2)$
PROP-ALL	$b_i \rightarrow (\alpha, \beta, c, \theta_1, \theta_2, \overline{q})$

computed on the biometric references (enrolment step), *iii*) search of a probe given its corresponding triangle characteristics (authentication step).

#### 3.1 Indexing based on triangular characteristics

Let  $\mathbf{T} = \{t_1, \dots, t_k\}$  be an unique set of minutiae triplets (*i.e.* triangle set) generated by an expanded Delaunay-based triangulation [Ka14] per fingerprint sample. Triangular characteristics are computed for each triangle: *i*) first and second largest angle,  $\alpha$  and  $\beta$  respectively; *ii*) the lateral length of the longest side, c; *iii*) first and second largest difference in ridge flow direction between two minutiae points,  $\theta_1$  and  $\theta_2$  respectively, *iv*) mean quality score  $\overline{q}$  of the three minutiae points that form the triangle, in the range [0,1]. Different types of triangular characteristic-based triplets are clustered in a bin  $b_i$  for each triangle per fingerprint sample, *e.g.*  $b_i = (\alpha, \beta, c)$ . Tab. 2 shows an overview on the different types of triplets (bins) which are evaluated in our work. Let  $\mathbf{R}$  be the enrolment database (*i.e.* the set of biometric references), this is organised as a Z-dimensional bin table where each dimension is corresponding to one characteristic within a triplet (*e.g.*  $Z_j \to \alpha$ ). Finally, we enrol each biometric reference (fingerprint) in  $\mathbf{R}$  by using its corresponding  $b_i$ . As expected, a same fingerprint can be enrolled on different  $b_i$  as this can describe different triangle characteristics for a same type of triplet according to their triangles generated.

#### 3.2 Retrieval by triangular characteristics

In the identification transaction, following similar steps from the indexing process, a set of triangular characteristics is computed and represented as a type of triplet  $(e.g.\ \alpha, \beta, c)$  from a probe. Note that a list of triplets for a same type  $(e.g.\ PROP1$  in Tab. 2) is generated from a probe. In order to find its corresponding biometric reference identifier, the scheme starts searching at the candidate lists retrieved per generated triplet until a match is found. In other words, each triplet built from a probe is indexed as a bin in  $\mathbf{R}$  to find the corresponding biometric identifier. Even, if the biometric identifier is not found from the set of triplets generated from the probe,  $i.e.\ each\ t_i \to b_i$ , then, the bins nearest to the triplet characteristics of the probe are visited in  $\mathbf{R}$  in order to find a match.

### 4 Experiments

In this section, the experimental setup and the used dataset are described (Sect. 4.1 and Sect. 4.2, respectively), along with the results of the experiments (Sect. 4.4).

#### 4.1 Experimental setup

In this section, we evaluate whether the minutiae quality information leads to the most stable triangular properties for indexing. To that end, expanded Delaunay-based triangulation is computed on different number of minutiae where minutiae quality is taken into account. In the context of minutiae quality, we compute continuous quality values per fingerprint sample based on a recently introduced approach: MiDeCon [Te21]. Data normalisation (min-max normalisation) is applied on the quality values. It is important to note that MiDeCon [Te21] represents the current state-of-the-art for minutiae quality estimation which considers the prediction confidence of the minutiae extractor [Te21]. Identification experiments are evaluated in a closed-set scenario, *i.e.* capture subjects envolved in a one-to-many comparisons are enrolled in the system. Therefore, for each fingerprint instance, a single sample was randomly selected as biometric reference and the remaining samples were used as probe samples in the search. Sub-sampling over 10 rounds is performed for all the experiments. Different search strategies as shown in Tab. 2 have been evaluated on the indexing scheme.

It is worth noting that a comparison with the state-of-the-art (Tab. 1) has not been analysed on this work, due to the different experimental protocols and databases evaluated. However, from Tab. 1 can be observed that WR schemes manage to be more competitive in their searches when the quality information is included.

#### 4.2 Databases

Experiments are conducted on the database FVC2006 [Ca07] which allows evaluating fingerprint samples captured with varying types of sensors, which is also expected to to lead to variations in minutiae qualities. This database consists of 7,200 fingerprint samples from 600 different fingerprints over four databases (DB): DB1 (electric field sensor), DB2 (optical sensor), DB3 (thermal sweeping sensor), and DB4 (synthetic images). In particular, in our experiments, we used the fingerprint samples for the testing set: 12 samples of 140 different fingerprints corresponding to different instances, for a total of 1,680 images per DB.

#### 4.3 Metrics

The biometric performance is evaluated in terms of metrics defined by ISO/IEC19795-1:2021 [IS21]: penetration rate (PR), number of identification transaction comparisons

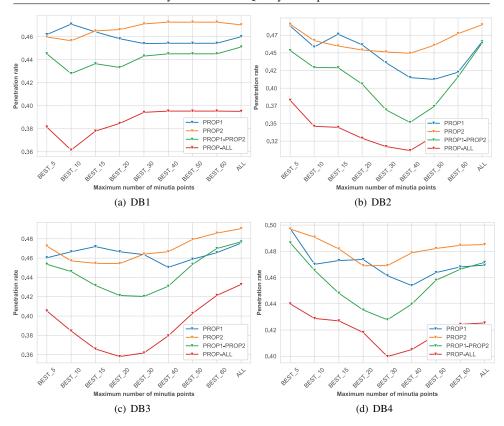


Fig. 3: Effect of the optimal number of minutiae ranked by its minutiae quality across different databases providing different types of sensor. Different indexing schemes have been evaluated taking into account different types of triples as described in Tab. 2. The label "ALL" include to all the minutiae points including different quality values.

(*i.e.* # of comparisons), and hit rate (HR). In addition, it is worth mentioning that the different indexing schemes (Tab. 2) evaluate WR applying two retrieval strategies, which are considered in the state-of-the-art for fingerprint indexing [Ma22]:

- **Incremental search**: bins in enrolment which are visited until a match is found. Thus, the workload baseline (*i.e.* exhaustive search) is approximately 50% for a set of identification transactions as the search will always find a match in the biometric reference set (*i.e.* HR=100%).
- **Fixed penetration**: the search is halted as soon as a match is found, or when a given maximum partition of the database (PR) has been explored. Hence, in this evaluation, the workload baseline is 100% for a set of identification transactions.

Tab. 3: Average number of comparisons and its corresponding penetration rate averaged for a set of identification transactions. Best results are shown for a fixed subset of minutiae points over different indexing strategies (*i.e.* type of triplet).

Database	Type of triplet	Optimal number of minutiae	# of comparisons	PR (%)
DB1	PROP1	30	$63.57 \pm 41.00$	45.41
	PROP2	10	$63.91 \pm 39.98$	45.65
	PROP1-PROP2	10	$59.92 \pm 39.44$	42.80
	PROP-ALL	10	$50.61 \pm 36.50$	36.15
DB2	PROP1	50	57.74± 40.95	41.24
	PROP2	40	$62.91 \pm 40.52$	44.94
	PROP1-PROP2	40	$49.21 \pm 39.38$	35.15
	PROP-ALL	40	$43.64 \pm 35.73$	31.17
DB3	PROP1	40	$63.06 \pm 40.74$	45.04
	PROP2	20	$63.60 \pm 40.00$	45.43
	PROP1-PROP2	30	$58.85 \pm 40.46$	42.03
	PROP-ALL	20	$50.13\pm37.13$	35.81
DB4	PROP1	40	$63.55 \pm 40.61$	45.39
	PROP2	20	$65.66 \pm 40.04$	46.90
	PROP1-PROP2	30	$59.89 \pm 39.39$	42.78
	PROP-ALL	30	$55.97 \pm 39.11$	39.98

#### 4.4 Results

Fig. 3 shows the average number of penetration rates empirically computed on different number of minutiae points in an incremental search. Tab. 3 shows the best results analysed from the Fig. 3. It should be noted that different numbers of minutiae are empirically selected according to their quality values ranked. Minutiae points are ranked from highest to lowest quality.

Taking a closer look at Fig. 3, we can perceive that the selection of the minutiae points corresponding to their ranked qualities has a significant impact on the system WR by reducing the workload baseline (PR<50%). It should be considered that any minutiae search process takes less consuming time over a reduced subset of minutiae than on the full set (*i.e.* "ALL"). In the context of workload in terms of penetration rates, better chances up to a certain number of minutiae can be noted across different sensors used per DB. Lowest penetration rates on average (*e.g.* PR = $\sim$ 34.38%) are achieved on the thermal, optical, and electrical sensors than on synthetic images (*e.g.* PR = $\sim$ 40%). However, it should be mentioned that DB4 should perceive better image quality [Te21], thus better minutiae quality. In addition, worst results for the PROP1 and PROP2 strategies w.r.t. PROP1-PROP2 and PROP-ALL, respectively are expected, as including more independent

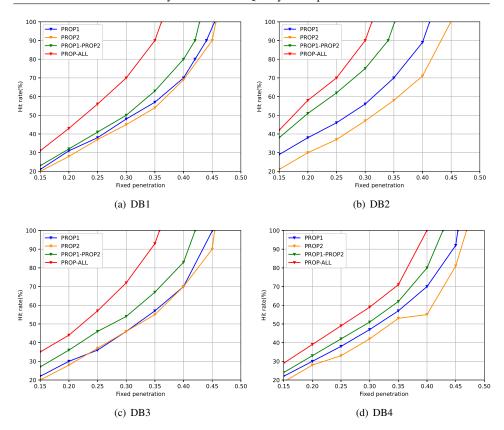


Fig. 4: Relation of the hit-rate across different fixed penetration rates. Results are shown for the best configurations of the Tab. 3 per indexing scheme and type of sensor.

properties decreases the likelihood that different minutia triplets have similar features by chance and are indexed into the same bin.

On the other hand, it is observed that including the average quality corresponding to the triangle minutiae points  $(\overline{q})$  as a feature in the triplet (*i.e.* **PROP-ALL**), penetration rates dropped drastically compared to the other indexing strategies analysed.

Finally, Fig. 4 shows the relation HR w.r.t different partitions (*i.e.* fixed penetration retrieval-based) of the enrolment database. As can be noted, the chance of getting a HR < 100% with lowest PR do not produce statistically significant results w.r.t. the incremental searches (Tab. 3). However, for the best configuration (PROP-ALL), it can be observed that 50% of the subjects can be found while guaranteeing a  $PR \le 25\%$  on average. Future works in this area may consider quality multi-stage-based search proposals. That is, fixed penetration searches-based by *e.g.* minutiae-quality, combined with incremental searches by *e.g.* image quality (see *e.g.* [Os22]), could reduce the overall workload in identification transactions.

#### 5 Conclusions

In this paper, the minutia quality has been analysed as a step prior to a minutiae point-based search scheme (*e.g.* search of minutiae triplets using the extended Delaunay triangulation) to reduce the number of comparisons per fingerprint identification transaction. More precisely, the variability of the minutiae qualities exhibited on fingerprint samples led to an improvement of the triangular properties for indexing, even, for a reduced subset of minutiae points. In addition, experimental results showed that the average minutiae quality as a criteria of the minutiae triangles should be considered for WR. Finally, it is expected that further works for WR in fingerprint identification are inspired to incorporate minutia quality prior to their minutiae indexing-based schemes.

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