Direct Detection and Location
of Multiple Sources with Intermittent Emission

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Abstract: This paper investigates the direct position determination (DPD) problem from passive measurements made with a moving antenna array in the case of a time-varying number of emitting sources. We derive the Cramér-Rao Bound (CRB) for the estimation problem and find an approximation that is applicable for a large number of observations. We use two DPD approaches to solve the estimation problem based on the Capon method and on the deterministic Maximum Likelihood (ML) estimator using a low- and high-dimensional optimization, respectively. The ML-DPD approach offers a superior performance compared to the Capon-DPD approach, but leads to a high-dimensional optimization. We use the Alternating Projection technique to solve the high-dimensional optimization by a sequence of low-dimensional optimizations. We propose an iterative approach that combines the source location and the determination of the total number of sources (referred as detection). We use a sequence of statistical tests to decide that the choice of the source number is correct. We present simulation results that demonstrate the performance of the method.

1   Introduction

Location of multiple narrowband sources using passive antenna arrays is a fundamental task encountered in various fields like wireless communication, radar, and sonar. We consider a scenario with a single moving observer equipped with an antenna array. At N different points in space the sensor receives signals of Q fixed sources and collects batches of antenna outputs. The scenario is assumed to be stationary during one batch and non-stationary from batch to batch.

According to the traditional approach to solving the location problem, first of all, the Directions of Arrival (DOAs) of all sources are estimated with a direction-finding (DF) estimator like the Capon method [Cap69] or the subspace-based Multiple Signal Classification (MUSIC) method [Sch86]. Typically, DF systems report measurements of different origin, e.g. true targets and clutter (false alarms). Then a bearing data association step follows to partition the DOAs into sets of DOAs belonging to the same source. Multiple Hypotheses Tracking (MHT) is generally accepted as the preferred method for solving the data association problem [Bla04]. In the last step, the DOAs for each source are used to estimate its position with the help of a suitable bearings-only tracking algorithm [Bec01].

Recently, some direct position determination (DPD) methods based on the antenna out-

puts have been proposed without computing intermediate parameters like DOAs. The basic idea for a subspace-based DPD approach goes back to the pioneering work of Wax and Kailath [WK85a]. They noted that in this way the data association step is avoided. Moreover, this kind of approach was used for a multiarray network in order to estimate the positions of multiple sources without explicitly computing DOAs and Times of Arrival (TOAs) [WA06]. ML methods can be found e.g. in [Wei04, AW07], but they are more computationally demanding in the case of multiple sources. The DPD approach can be adapted to estimate DOAs and DOA rates [WE95].

In our previous work, we proposed a subspace-based DPD approach (referred to as Subspace Data Fusion (SDF)) for a single moving array [DOR08]. Moreover, we have shown that the DPD approach can be extended to estimate the target states (e.g. positions, velocities) [OD08] and to the multisensor case to avoid the track-to-track association problem [Ois09a]. Furthermore, we adapt the DPD approach to solve the bearing data association problem in the presence of clutter by using a fictitious array [ODW08]. In all these DPD approaches, the parameters of interest are obtained by minimizing a single cost function into which all array batches enter jointly (Fig. 1). Moreover, the estimation accuracy of the source position is much better than the traditional location approach in situations where the variance of DOA estimates deviates from the corresponding Cramér-Rao Bound (CRB).

In [Ois09b], we investigated the case where the number of emitting sources is time-varying. Farina *et al.* derived the CRB for the general case that the probability of detection is smaller than unity [FRT02]. We extended the results to the case of multiple sources with intermittent emission and adapted them to derive the CRB for the direct state determination problem. We used a SDF approach based on the MUSIC method [Sch86] and proposed an extension by using the Subspace Fitting (SSF) method described in [VO91]. We have shown that the state estimation accuracy of the SSF-SDF approach is much better compared to the MUSIC-SDF approach in situations of a time-varying number of emitting sources.

In this paper, we present two DPD approaches based on the Capon method [Cap69] and the deterministic Maximum Likelihood (ML) estimator. These approaches use the full data covariance matrices instead of the corresponding subspaces. The ML-DPD approach offers a superior performance in comparison to the Capon-DPD approach, but leads to a high-dimensional optimization. We use the Alternating Projection (AP) technique to solve the high-dimensional optimization by a sequence of low-dimensional optimizations
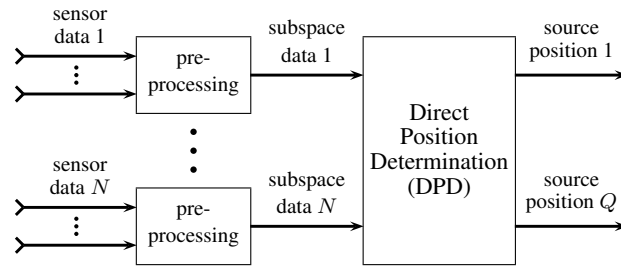

Figure 1: Basic steps of the DPD approach

[ZW88] and we find similar results for the ML-DPD approach and the AP-DPD approach. The DPD approaches require knowledge of the total number of sources, but in practice this number is unknown. We propose an iterative solution that combines the source location and the determination of the total number of sources (in the field of array signal processing referred to as detection). We use a sequence of statistical tests to decide that the choice of the source number is correct. The basis of the tests is the fact that the optima of the ML-DPD cost function are χ^2-distributed. We present simulation results that demonstrate the performance of the method.

This paper is organized as follows: Section 2 introduces the concept of emitting/non-emitting sequences, Section 2.1 presents the data model, and Section 2.2 formulates the estimation problem. In Section 2.3 we derive the CRB for the described DPD problem and find an approximation which is applicable for practical purposes. In Section 3.1 and Section 3.2, we outline the considered Capon-DPD approach and ML-DPD approach, in Section 3.3 we compare both approaches in simulations, and in Section 3.4 we present the AP technique to solve the high-dimensional optimization with a lower computational complexity. In Section 4 we propose a combined detection and location approach. The conclusions are given in Section 5.

The following notations are used throughout this paper: $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively; $\mathbf{I}_n$ and $\mathbf{0}_n$ denote the $n \times n$-dimensional identity and zero matrix, respectively; and $\mathrm{E}\{\cdot\}$ denotes the expectation operation.

2 Estimation Problem

For N observations, 2^N possible emitting/non-emitting sequences per source can be formed (Fig. 2). The κ-th possible sequence reads $S_{q,\kappa} : (b_{1,q})_\kappa, ..., (b_{N,q})_\kappa, \kappa = 1, ..., 2^N$, where $b_{n,q}$ is a binary variable that corresponds to the case where the q-th source, $q = 1, ..., Q$, is emitting or non-emitting at time t_n, $n = 1, ..., N$. For a given emitting probability
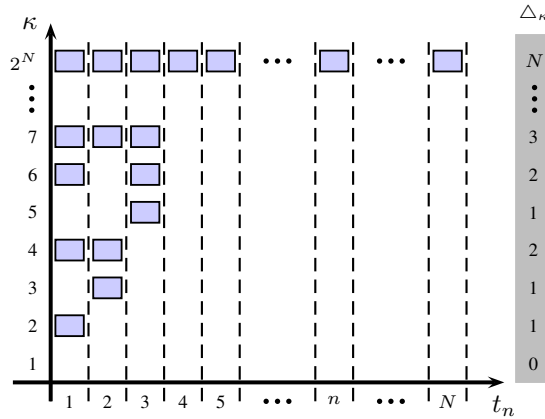

Figure 2: List of all possible emitted sequences

$P_{\mathrm{e},q}$, which is constant over the number of observations, the probability of occurrence of a particular emitting/non-emitting sequence is given by

$$P(S_{q,\kappa}) = P_{\mathrm{e},q}^{\triangle_\kappa} \left(1 - P_{\mathrm{e},q}\right)^{\bar{\triangle}_\kappa}, \tag{1}$$

where $\triangle_\kappa$ and $\bar{\triangle}_\kappa = N - \triangle_\kappa$ are the number of observations where the q-th source does emit or does not emit, respectively. For Q sources, 2^{NQ} collections of independent emitting/non-emitting sequences called events are possible. The probability of occurrence of a particular event $E_\ell : (\kappa_1)_\ell, ..., (\kappa_Q)_\ell, \ell = 1, ..., 2^{NQ}$, is given by

$$P(E_\ell) = \prod_{q=1}^{Q} P(S_{q,(\kappa_q)_\ell}). \tag{2}$$

To illustrate with a simple example the nature of Eq. 1 and Eq. 2, we consider just four observations of a single target. The emitting/non-emitting sequence $(1, 1, 0, 1)$ has probability of occurrence $P_{\mathrm{e},1}^3 \left(1 - P_{\mathrm{e},1}\right)$. For a second target with the sequence $(1, 1, 1, 1)$, the corresponding event has probability of occurrence $P_{\mathrm{e},1}^3 \left(1 - P_{\mathrm{e},1}\right) P_{\mathrm{e},2}^4$. Similar expressions can be written for the remaining events.

2.1 Data Model

We consider an antenna array composed of M elements mounted on a moving platform and Q fixed sources located at $\mathbf{x}_q = (x_q, y_q, z_q)^T$ in the far field of the antenna array. The sources are assumed to radiate narrowband signals (i.e. the source bandwidth is much smaller than the reciprocal of the time delay across the array) with wavelengths centered around a common wavelength λ. The sensor moves along an arbitrary but known trajectory (Fig. 3). During the movement of the array, N batches of data are collected at the positions $\mathbf{r}_n, n = 1, ..., N$. For the sake of simplicity, we assume that the antenna attitude does not change with time, i.e. the orientation of the sensor-fixed coordinate system is fixed during the batches. The distance between the q-th source and the observer at the n-th time slot, $\triangle r_{n,q}$, is given by the length of the relative vector

$$\triangle \mathbf{r}_n(\mathbf{x}_q) = \mathbf{r}_n - \mathbf{x}_q. \tag{3}$$

Let $s_{n,k,q}$ denote the complex envelope of the k-th sample, $k = 1, ..., K$, of the q-th source signal measured at time t_n if this source emits, i.e. $b_{n,q} = 1$, and let $\mathbf{z}_{n,k} \in \mathbb{C}^{M \times 1}$ denote the complex envelopes formed from the signals received by the array elements. This received vector can be expressed as

$$\mathbf{z}_{n,k} = \sum_{q=1}^{Q} \mathbf{a}_n(\mathbf{x}_q) \, b_{n,q} \, s_{n,k,q} + \mathbf{w}_{n,k}, \tag{4}$$

where $\mathbf{w}_{n,k} \in \mathbb{C}^{M \times 1}$ is the complex envelope of the noise. Let the array be sampled sequentially at K different mutually exclusive time slots, and assume that the array transfer
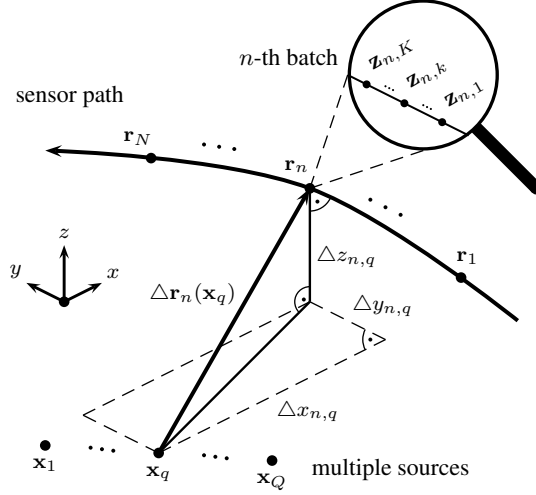
Figure 3: Geometry for the scenario of multiple inertially moving sources and a single moving sensor

vectors can be considered quasistatic in each slot, i.e. the sensor's displacement during each time slot is negligible. The array transfer vector expresses its complex response at time t_n to a planar wavefront arriving from the direction of the relative position $\triangle \mathbf{r}_n(\mathbf{x}_q)$ (Eq. 3). External and blind array calibration techniques are well-known, e.g. the calibration of an airborne antenna array is described in [MSHK07]. We assume that the antenna array is perfectly calibrated for which the array transfer vector is a known function of the source positions:

$$\mathbf{a}_n(\mathbf{x}_q) = \left(e^{j\,\mathbf{k}_n^T(\mathbf{x}_q)\,\mathbf{d}_1}, ..., e^{j\,\mathbf{k}_n^T(\mathbf{x}_q)\,\mathbf{d}_M} \right)^T \tag{5}$$

The array transfer vector depends on the position $\mathbf{d}_m$ of the m-th antenna element, $m = 1, ..., M$, relative to the position $\mathbf{r}_n$, and the wavenumber vector

$$\mathbf{k}_n(\mathbf{x}_q) = \frac{2\pi}{\lambda} \frac{\triangle \mathbf{r}_n(\mathbf{x}_q)}{\triangle r_{n,q}} . \tag{6}$$

Eq. 4 can be written more compactly as

$$\mathbf{z}_{n,k} = \mathbf{A}_n(\boldsymbol{\rho}_{\mathbf{x},n})\,\check{\mathbf{s}}_{n,k} + \mathbf{w}_{n,k} , \tag{7}$$

where $\mathbf{A}_n(\boldsymbol{\rho}_{\mathbf{x},n}) = [\mathbf{a}_n(\mathbf{x}_1) \cdots \mathbf{a}_n(\mathbf{x}_{Q_n})] \in \mathbb{C}^{M \times Q_n}$ is the array transfer matrix, and

$$\boldsymbol{\rho}_{\mathbf{x},n} = (\mathbf{x}_1^T, ..., \mathbf{x}_{Q_n}^T)^T \in \mathbb{R}^{3Q_n \times 1}$$
$$\check{\mathbf{s}}_{n,k} = (s_{n,k,1}, ..., s_{n,k,Q_n})^T \in \mathbb{C}^{Q_n \times 1}$$

denote subsets from the complete parameter vectors

$$\boldsymbol{\rho}_{\mathbf{x}} = (\mathbf{x}_1^T, ..., \mathbf{x}_Q^T)^T \in \mathbb{R}^{3Q \times 1}$$

$$\mathbf{s}_{n,k} = (s_{n,k,1}, ..., s_{n,k,Q})^T \in \mathbb{C}^{Q \times 1} \tag{8}$$

w.r.t. the effective number of emitting sources $Q_n = \sum_{q=1}^{Q} b_{n,q}$ at the n-th batch.

Now, we introduce the compact data model

$$\mathbf{z}_k = \boldsymbol{\mathcal{A}}(\boldsymbol{\rho}_{\mathbf{x}}) \check{\mathbf{s}}_k + \mathbf{w}_k \tag{9}$$

by stacking the vectors on top and using a block-diagonal matrix:

$$
\begin{aligned}
\mathbf{z}_k &= (\mathbf{z}_{1,k}^T, ..., \mathbf{z}_{N,k}^T)^T \in \mathbb{C}^{MN \times 1}, \\
\boldsymbol{\mathcal{A}}(\boldsymbol{\rho}_{\mathbf{x}}) &= \mathrm{diag}[\mathbf{A}_1(\boldsymbol{\rho}_{\mathbf{x},1}) \cdots \mathbf{A}_N(\boldsymbol{\rho}_{\mathbf{x},N})] \in \mathbb{C}^{MN \times \sum_n Q_n}, \\
\check{\mathbf{s}}_k &= (\check{\mathbf{s}}_{1,k}^T, ..., \check{\mathbf{s}}_{N,k}^T)^T \in \mathbb{C}^{\sum_n Q_n \times 1}, \\
\mathbf{w}_k &= (\mathbf{w}_{1,k}^T, ..., \mathbf{w}_{N,k}^T)^T \in \mathbb{C}^{MN \times 1}.
\end{aligned}
$$

2.2 Problem Formulation

The received data batches depend on the array transfer vectors, which depend on the relative vectors, which themselves depend on the desired source positions. Now, the problem is stated as follows: Estimate all source positions $\boldsymbol{\rho}_{\mathbf{x}}$ from all received signals $\mathbf{Z}_n = [\mathbf{z}_{n,1}, ..., \mathbf{z}_{n,K}] \in \mathbb{C}^{M \times K}$, $n = 1, ..., N$. To solve the DPD problem, the following assumptions are made:

A1. The noise vectors $\mathbf{w}_k$, $k = 1, ..., K$, (Eq. 9) are zero-mean complex Gaussian. They are temporally and spatially uncorrelated with the covariance

$$
\begin{aligned}
\mathrm{E}\left\{\mathbf{w}_k \mathbf{w}_{k'}^H\right\} &= \sigma_{\mathrm{w}}^2 \mathbf{I}_{MN} \delta_{k,k'}, \\
\mathrm{E}\left\{\mathbf{w}_k \mathbf{w}_{k'}^T\right\} &= \mathbf{0}_{MN},
\end{aligned} \tag{10}
$$

where $\delta_{k,k'}$ denotes the Kronecker delta.

A2. The signal vectors $\check{\mathbf{s}}_{n,k}$, $n = 1, ..., N$, $k = 1, ..., K$, (Eq. 7) are fixed and need to be estimated (deterministic data model). This does not exclude the possibility that the signals are sampled from a random process.

A3. The effective number of sources per batch Q_n, $n = 1, ..., N$, is time-varying but known. In the past, several methods have been proposed to determine the source number, e.g. in [WK85b]. In Section 3, we assume that the total number of sources Q is known, and in Section 4, Q is unknown.

2.3 Cramér-Rao Bound

For judging an estimation problem, it is important to know the maximum estimation accuracy that can be attained with all given measurements $\mathbf{Z}$. It is well known that the CRB

provides a lower bound on the estimation accuracy for any unbiased estimator $\hat{\boldsymbol{\rho}}(\mathbf{Z})$ and its parameter dependencies reveal characteristic features of the estimation problem. Given a particular event E_ℓ, the target parameters are comprised in the vector

$$\boldsymbol{\rho} = \left(\bar{\tilde{\mathbf{s}}}_1^T, \tilde{\mathbf{s}}_1^T, ..., \bar{\tilde{\mathbf{s}}}_K^T, \tilde{\mathbf{s}}_K^T, \boldsymbol{\rho}_{\mathbf{x}}^T\right)^T \in \mathbb{R}^{2K\sum_n Q_n + 3Q \times 1}, \tag{11}$$

where overbar and overtilde are the real and imaginary part of the source signals. Then, the conditional CRB is related to the covariance matrix $\mathbf{C}$ of the estimation error $\triangle\boldsymbol{\rho} = \boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\mathbf{Z})$ as

$$\mathbf{C} = \mathrm{E}\left\{\triangle\boldsymbol{\rho}\,\triangle\boldsymbol{\rho}^T | E_\ell\right\} \geq \mathrm{CRB}(\boldsymbol{\rho}|E_\ell), \tag{12}$$

where the inequality means that the matrix difference is positive semidefinite. If the estimator attains the CRB then it is called efficient. The CRB is given by the inverse Fisher Information Matrix (FIM)

$$\mathbf{J}(\boldsymbol{\rho}|E_\ell) = \mathrm{E}\left\{ \left(\frac{\partial \mathcal{L}(\mathbf{Z}; \boldsymbol{\rho})}{\partial \boldsymbol{\rho}}\right) \left(\frac{\partial \mathcal{L}(\mathbf{Z}; \boldsymbol{\rho})}{\partial \boldsymbol{\rho}}\right)^T \middle| E_\ell \right\}, \tag{13}$$

where

$$\mathcal{L}(\mathbf{Z}; \boldsymbol{\rho}) = -KMN \ln(\pi\sigma_{\mathrm{w}}^2) - \frac{1}{\sigma_{\mathrm{w}}^2} \sum_{k=1}^{K} |\mathbf{z}_k - \boldsymbol{\mathcal{A}}(\boldsymbol{\rho}_{\mathbf{x}})\,\check{\mathbf{s}}_k|^2, \tag{14}$$

is the log-likelihood function and the parameters refers to the event E_ℓ. In this log-likelihood function $\mathbf{z}_k$, $k = 1, ..., K$, are random variables due to the random variables $\mathbf{w}_k$, $k = 1, ..., K$, and the expectation operation in Eq. 13 is w.r.t. these random variables.

Performing all calculations analog to [Ois09b, SN89, YB92], we obtain the deterministic CRB for all source positions after some algebra (Assumption A1):

$$\mathrm{CRB}(\boldsymbol{\rho}_{\mathbf{x}}|E_\ell) = \frac{\sigma_{\mathrm{w}}^2}{2} \left[\sum_{k=1}^{K} \mathrm{Re}\left\{ \boldsymbol{\mathcal{S}}_k^H \mathbf{D}^H \mathbf{P}_{\boldsymbol{\mathcal{A}}}^{\perp} \mathbf{D}\,\boldsymbol{\mathcal{S}}_k \right\} \right]^{-1} \tag{15}$$

with

$$\begin{aligned}
\boldsymbol{\mathcal{S}}_k &= \mathbf{I}_{3Q} \otimes \check{\mathbf{s}}_k \in \mathbb{C}^{3Q\sum_n Q_n \times 3Q}, \\
\mathbf{D} &= \left[\frac{\partial \boldsymbol{\mathcal{A}}}{\partial x_1}, \frac{\partial \boldsymbol{\mathcal{A}}}{\partial y_1}, \frac{\partial \boldsymbol{\mathcal{A}}}{\partial z_1}, \cdots, \frac{\partial \boldsymbol{\mathcal{A}}}{\partial x_Q}, \frac{\partial \boldsymbol{\mathcal{A}}}{\partial y_Q}, \frac{\partial \boldsymbol{\mathcal{A}}}{\partial z_Q} \right] \in \mathbb{C}^{MN \times 3Q\sum_n Q_n}, \\
\mathbf{P}_{\boldsymbol{\mathcal{A}}}^{\perp} &= \mathbf{I}_{MN} - \boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{A}}^H \boldsymbol{\mathcal{A}})^{-1}\boldsymbol{\mathcal{A}}^H \in \mathbb{C}^{MN \times MN},
\end{aligned}$$

where $\otimes$ denotes the Kronecker product.

The bound in Eq. 15 is conditioned on the particular event E_ℓ. The unconditional CRB is obtained by taking expectation and using Eq. 2

$$\mathrm{CRB}(\boldsymbol{\rho_x}) = \sum_{\ell=1}^{2^{NQ}} P(E_\ell)\,\mathrm{CRB}(\boldsymbol{\rho_x}|E_\ell)\,. \tag{16}$$

Observe that although the number of possible events grows exponentially with the number of batches N and sources Q, the probabilities of the vast majority of events (Eq. 2) are negligible.

The cumulative distribution function (cdf) ϕ_q of the number of batches $\bar{\triangle} = 0, ..., N$ with the non-emitting q-th source is given by [FRT02, Eq. 23]

$$\phi_q(\bar{\triangle}) = \sum_{\bar{\delta}=0}^{\bar{\triangle}} \binom{N}{\bar{\delta}} P_{\mathrm{e},q}^{N-\bar{\delta}} (1 - P_{\mathrm{e},q})^{\bar{\delta}}\,. \tag{17}$$

By definition, $\phi_q(N) = 1$. Above a certain threshold value $\bar{\triangle}_{\mathrm{thr},q}$, all events corresponding to this sequences can be safely ignored in the calculation of the CRB in Eq. 16 for all practical purposes. The threshold value $\bar{\triangle}_{\mathrm{thr},q}$ can be determined by progressively computing Eq. 17 for $\bar{\triangle} = 0, 1, 2, ...$ until its value is greater than some cdf threshold ϕ_{thr}, which should be chosen to be marginally less than 1, e.g. $\phi_{\mathrm{thr}} = 0.99$, i.e. [FRT02, Eq. 24]

$$\bar{\triangle}_{\mathrm{thr},q} = \min\{\bar{\triangle}\,,\ \mathrm{s.t.}\ \phi_q(\bar{\triangle}) > \phi_{\mathrm{thr}}\}\,. \tag{18}$$

This strategy ensures that only events are considered which contribute significantly. Eq. 19 gives the number of events L_{approx} to take into account in the approximate calculation of the CRB (Eq. 16). A reduction of ϕ_{thr} would correspond to less computational load but also a reduced accuracy.

$$L_{\mathrm{approx}} = \prod_{q=1}^{Q} \sum_{\bar{\delta}=0}^{\bar{\triangle}_{\mathrm{thr},q}} \binom{N}{\bar{\delta}} \ll 2^{NQ} \tag{19}$$

As an example, we consider two sources ($Q = 2$) with the emitting probabilities $P_{\mathrm{e},1} = 0.9$ and $P_{\mathrm{e},2} = 0.7$, and $N = 12$ observations. Then, the total number of all possible events is $2^{NQ} = 16,777,216$. Fig. 4 displays the cdf (Eq. 17) of the misses, namely $\bar{\triangle}$. For the first and the second source are sequences with more than 4 or 7 misses, respectively, very unlikely. Consequently, this sequences can be ignored and the number of sequences is reduced 794 or 3302 for the first and second source, respectively. Finally, we take $L_{\mathrm{approx}} = 2,621,788$ events into account to evaluate Eq. 17.

3   Direct Location

In this section, we outline the DPD approaches (Fig. 1) to solving the location problem with the assumption that the number of sources Q is known. This approach relies on the
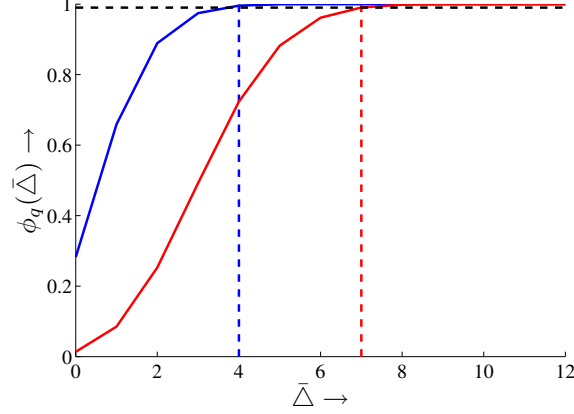
Figure 4: Cumulative distribution function for $Q = 2$ sources and $N = 12$ observations; source 1 (blue lines), source 2 (red lines)

same key idea as the localization approach of Wax and Kailath for decentralized array processing [WK85a]. They mentioned that this kind of estimation offers the advantage that the association problem inherent to the traditional method is circumvented. Furthermore, no intermediate parameters like DOAs or additional parameters like DOA variances are necessary. Note that the proposed DPD approaches do not use knowledge about the emitting probabilities $P_{e,q}$, because they are not sensor parameters like the probability of detection.

The DPD approaches calculate the source positions directly in one step from the full covariance matrices at all sensor positions $\mathbf{r}_n$, $n = 1, ..., N$, (Fig. 1):

$$\mathbf{R}_n = \frac{1}{K} \mathbf{Z}_n \mathbf{Z}_n^H .\tag{20}$$

In our previous work [Ois09b], we used similar approaches that calculate the source positions from the corresponding subspace data (referred to as SDF). We made the assumption that $\mathbf{R}_n^{-1} \approx \mathbf{U}_n \mathbf{U}_n^H$, where $\mathbf{U}_n \in \mathbb{C}^{M \times M - Q_n}$ are the eigenvectors spanning the noise subspace of the covariance $\mathbf{R}_n$.

3.1 Capon Approach

This DPD approach uses a Capon-type cost function [Cap69], which minimizes the sum of all projections of the array transfer vectors at the sensor positions onto the corresponding noise subspaces. The source positions are calculated directly in one step by fusing the covariances of all batches of all sensors:

$$f_{\text{Capon-DPD}}(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{a}_n^H(\mathbf{x}) \, \mathbf{R}_n^{-1} \, \mathbf{a}_n(\mathbf{x}) ,\tag{21}$$

where the array transfer vector (Eq. 5) is parameterized by the source position $\mathbf{x}$. The cost function shows minima for a proper choice of $\mathbf{x}$, if the covariance matrix of each batch is consistent with to the corresponding array transfer vector.

3.2 Maximum Likelihood Approach

This DPD approach is based on the same sequence of covariance matrices. Since the source signals $\check{\mathbf{s}}_k$, $k = 1, ..., K$, need to be estimated, we fix $\boldsymbol{\rho}_{\mathbf{x},n}$ and minimize Eq. 14 w.r.t. the signals. Substituting the well-known result back into Eq. 14, we obtain the cost function of the deterministic ML estimator, which obtains the array transfer matrix $\mathbf{A}_n(\boldsymbol{\rho}_{\mathbf{x}})$ to the measurements in a least squares sense by minimizing

$$f_{\text{ML-DPD}}(\boldsymbol{\rho}_{\mathbf{x}}) = \frac{2K}{\sigma_{\text{w}}^2} \sum_{n=1}^{N} \text{tr} \left\{ \mathbf{P}_{\mathbf{A}_n(\boldsymbol{\rho}_{\mathbf{x}})}^{\perp} \mathbf{R}_n \right\} , \tag{22}$$

where $\text{tr}\{\cdot\}$ denotes the trace operation and

$$\mathbf{P}_{\mathbf{X}}^{\perp} = \mathbf{I}_M - \mathbf{X} (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \tag{23}$$

is a $M \times M$-dimensional projection matrix that projects onto the column space of $\mathbf{X}$. This leads to a single search in $3Q$ dimensions instead of Q searches in 3 dimensions for the DPD approach (Assumption 3), but there are more degrees of freedom available for fitting.

3.3 Comparison of the DPD Approaches

As an illustration, we consider the DPD problem and a scenario in which the sensor moves along an arc from $(-0.5, -0.5, 0.5)^T$ km to $(0.5, -0.5, 0.5)^T$ km. Two sources are located on the ground at the positions $\mathbf{x}_1 = (0, -0.5, 0)^T$ km and $\mathbf{x}_2 = (0, 0.5, 0)^T$ km (Fig. 5, upper left). Furthermore, we consider a 10-element uniform circular array with element positions $\mathbf{d}_m = \rho \left(\cos \frac{m\pi}{5}, \sin \frac{m\pi}{5}, 0 \right)^T$ and radius $\rho = \frac{\lambda}{2} (\sin \frac{\pi}{10})^{-1}$.

With the assumption that the sensor lies always above each source ($\triangle z_{n,q} > 0$, $n = 1, ..., N$, $q = 1, ..., Q$), the considered problem has a unique solution, because the condition for unique DF of narrowband sources holds, which implies that $Q < M$ [WZ89], and the observability condition established in [Bec93] is satisfied.

Moreover, we assume $N = 12$ batches with $K = 100$ samples per batch. For the emitted waveforms of each source we assume that they have constant amplitude at the sensor positions: $|s_{n,k,q}| = s$, and we define the signal-to-noise ratio of a single source and single element: $\text{SNR} = s^2 / \sigma_{\text{w}}^2$.

Fig. 5 compares the cost functions of the DPD approaches for a fixed z-coordinate, and $\text{SNR} = 0$ dB. Furthermore, in the lower right plot the coordinates of the first source are fixed. In the upper right plot, we assume an emitting probability of $P_{\text{e},1} = P_{\text{e},2} = 1$. The
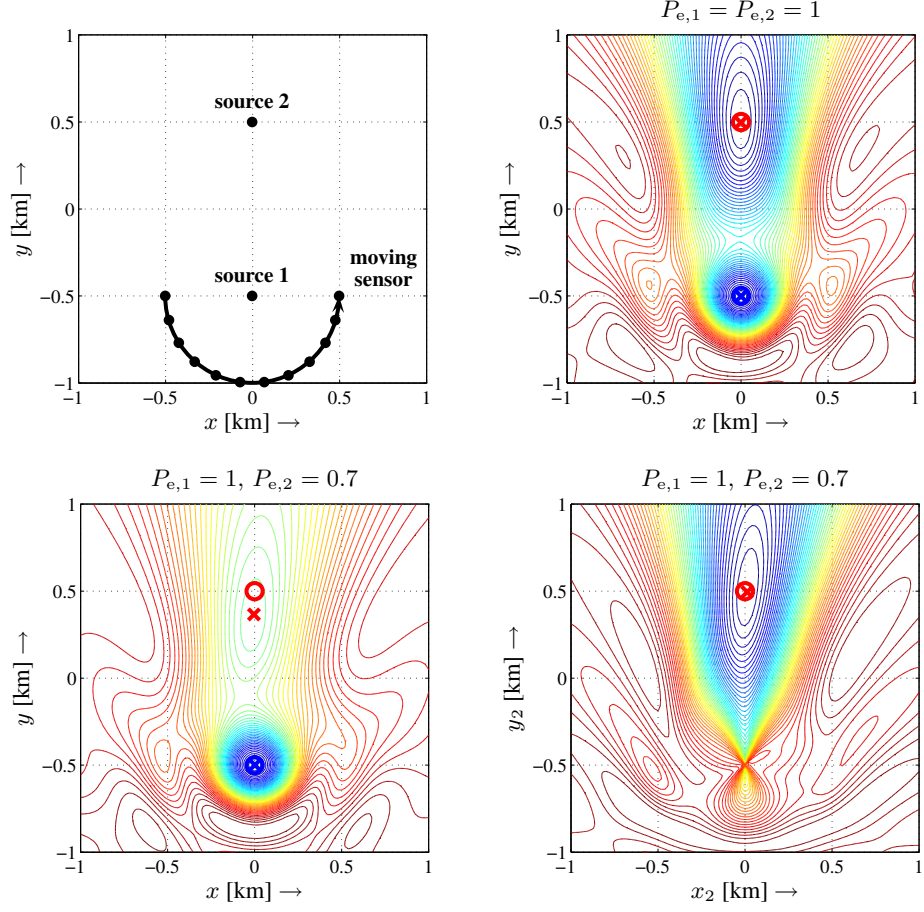
Figure 5: Upper left: Considered scenario; Upper right, lower left: xy-plane of the MUSIC-DPD cost function; Lower right: ML-DPD cost function with the true (circles) and estimated (crosses) position parameters for the given emitting probabilities

Capon-DPD cost function displays well-pronounced minima at the true target positions and no further local minima. In the following cases, the emitting probability of the second source is reduced to $P_{e,2} = 0.7$. Then, the Capon-DPD cost function introduces significant errors (lower left plot), while the ML-DPD cost function can account for missing signals (lower right plot). Note that for some events the Capon-DPD cost function displays no minimum at the location of the second source.

3.4 Alternating Projection Technique

The solution of the deterministic ML approach in Section 3.2 can be found by adapting the Alternating Projection (AP) algorithm described in [ZW88] from the DF problem to the direct location problem. The iterative AP technique is a simple technique for multidimensional optimization. Therein, at every iteration a minimization is performed w.r.t. a single parameter while all other parameters are held fixed. To solve the DPD problem, we held all parameters of a single source fixed (Fig. 6). Then, the AP-DPD algorithm performs the $3Q$-dimensional optimization by optimizing a sequence of 3-dimensional cost functions (Assumption 3). The q-th source position estimate at the $(i+1)$-th iteration, $\hat{\mathbf{x}}_q^{i+1}$, is obtained by minimizing

$$f_{\text{AP-DPD}}(\mathbf{x}_q) = \frac{2K}{\sigma_{\text{w}}^2} \sum_{n=1}^{N} \text{tr} \left\{ \mathbf{P}_{[\mathbf{A}_n(\hat{\boldsymbol{\rho}}_{\mathbf{x},\neg q}^i),\mathbf{a}_n(\mathbf{x}_q)]}^{\perp} \mathbf{R}_n \right\} \tag{24}$$

where $\hat{\boldsymbol{\rho}}_{\mathbf{x},\neg q}^i$ is a $3(Q-1) \times 1$-dimensional vector of the computed parameters in the i-th iteration step, but without the parameters of the q-th source:

$$\hat{\boldsymbol{\rho}}_{\mathbf{x},\neg q}^i = (\hat{\mathbf{x}}_1^{i\,T}, ..., \hat{\mathbf{x}}_{q-1}^{i\,T}, \hat{\mathbf{x}}_{q+1}^{i\,T}, ..., \hat{\mathbf{x}}_Q^{i\,T})^T . \tag{25}$$

We find the same performance for the ML-DPD and the AP-DPD approach (compare Fig. 5 (lower right) and Fig. 6 (right)).

As the initialization is critical for the global convergence, we start by minimizing Eq. 22 for a single source: $\mathbf{A}_n(\mathbf{x}_1^0) = \mathbf{a}_n(\mathbf{x}_1^0)$. Next, we solve Eq. 22 for the second source, assuming the first source location is at $\hat{\mathbf{x}}_1^0$: $\mathbf{A}_n(\mathbf{x}_2^0) = [\mathbf{a}_n(\hat{\mathbf{x}}_1^0), \mathbf{a}_n(\mathbf{x}_2^0)]$. Finally, all initial values $\hat{\mathbf{x}}_q^0$, $q = 1, ..., Q$, are computed by continuing in this fashion [ZW88].
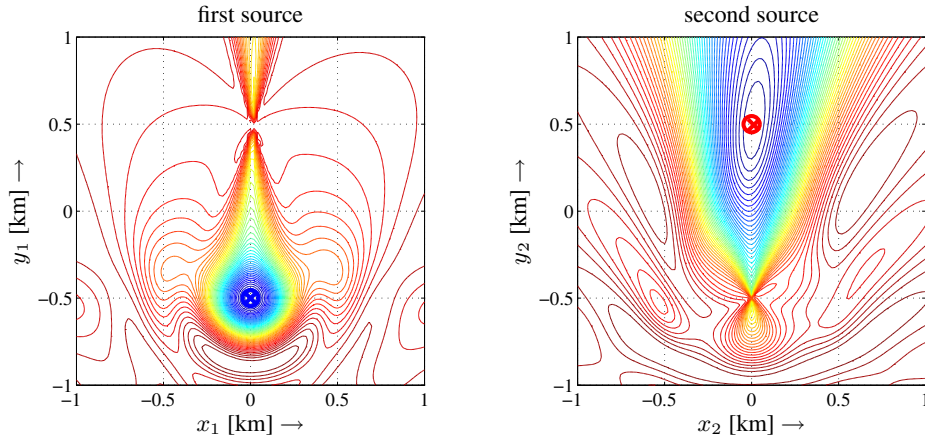

Figure 6: Cost functions of the AP-DPD for both sources in the case $P_{\text{e},1} = 1$ and $P_{\text{e},2} = 0.7$

4 Direct Detection and Location

In practice, the total number of sources Q is unknown, but the proposed approach in Section 3 requires the knowledge of Q in order to choose the correct dimension of $\boldsymbol{\rho}_{\mathbf{x}}$. A statistical test is necessary, to decide that the choice of $\hat{Q}$ is correct. The basis of the test proposed here is the following observation which is similar to the result in [VOK91, Sec. IV] for the DF problem: The quantity $f_{\text{ML-DPD}}(\boldsymbol{\rho}_{\mathbf{x}})$ (Eq. 22) is χ_ν^2-distributed with $\nu = 2K \sum_{n=1}^{N}(M - Q_n)$ degrees of freedom. For many degrees of freedom ($\nu > 30$), the χ_ν^2-distribution can be approximated by a normal distribution with mean ν and variance 2ν. We perform a sequence of tests increasing the number of sources $\hat{Q} = Q_0, ..., M$ starting with some value Q_0, e.g. $Q_0 = 1$. If

$$f_{\text{ML-DPD}}(\hat{\boldsymbol{\rho}}_{\mathbf{x}}) \leq \chi_{\alpha;\nu}^2 \quad , \hat{\boldsymbol{\rho}}_{\mathbf{x}} \in \mathbb{R}^{\hat{Q} \times 1} \tag{26}$$

holds, i.e. if the result of Eq. 22 is consistent with the expected χ_ν^2-distribution, then we stop, otherwise we increase the number of sources $\hat{Q}$. The quantity $\chi_{\alpha;\nu}^2$ is the α-percentage point of the χ_ν^2-distribution. The probability to overestimate the number of sources, i.e. the probability of false alarm, will be equal to α. We keep testing until Eq. 22 fits to the expected χ_ν^2-distribution. The case of an imperfect detection leads to false targets ($\hat{Q} > Q$) or misses ($\hat{Q} < Q$). Generally false targets are preferred.

For the scenario considered in the previous sections, Monte Carlo simulations with 1000 runs have been carried out to study the performance of the proposed approach. In approx. 99% of the cases, we find the true number for all considered SNR values, but for $\text{SNR} = -10$ dB, we underestimate the source number in approx. 40% of the cases due to resolution conflicts. In Fig. 7, we compare the root mean square error (RMSE) of the
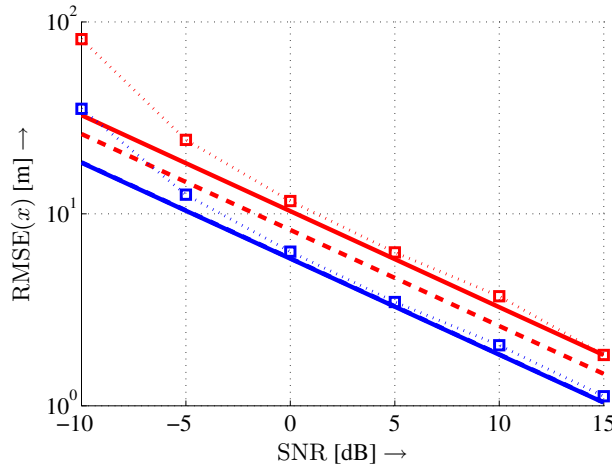

Figure 7: Square-root of the CRB for the considered $P_{\text{e},q}$ (solid lines) and $P_{\text{e},q} = 1$ (dashed lines) and the RMSE for the proposed approach (dotted lines) versus SNR for x-coordinate; source 1 (blue), source 2 (red)

estimates with the approximation of the CRB (Eq. 16). The RMSE attains the CRB for a high SNR and degrades slightly for weak sources.

5 Conclusions

We investigated the DPD problem for multiple sources with intermittent emission and proposed direct location approaches to solving the estimation problem. We summarize the content of the paper as follows:

1. We derived the deterministic CRB and presented a computationally convenient approximation which is applicable for practical purposes.

2. In simulations, we demonstrated the superior performance of the Capon-DPD approach compared to the ML-DPD approach.

3. We used the AP technique to solve the high-dimensional optimization by a sequence of low-dimensional optimizations. We find similar results for the ML-DPD approach and the AP-DPD approach. Furthermore, we present an initialization strategy that can be applied to the considered DPD approaches.

4. We proposed an iterative direct detection and location approach to determine the number of targets *and* the corresponding target locations. We investigate the proposed method in simulations.

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