

The Home Marking Problem and Some Related Concepts

Roxana Melinte Olivia Oanea Ioana Olga
Ferucio Laurențiu Țiplea

Faculty of Computer Science
“Al. I. Cuza” University
6600 Iași, Romania
e-mail: {abur,olivia,olgai,fltiplea}@infoiasi.ro

Abstract: In this paper we study the *home marking problem* for Petri nets, and some related concepts to it like confluence, noetherianity, and state space inclusion. We show that the home marking problem for inhibitor Petri nets is undecidable. We relate then the existence of home markings to confluence and noetherianity and prove that confluent and noetherian Petri nets have an unique home marking. Finally, we define some versions of the state space inclusion problem related to the home marking and sub-marking problems, and discuss their decidability status.

1 Introduction and Preliminaries

A *home marking* of a system is a marking which is reachable from every reachable marking in the system. The identification of home markings is an important issue in system design and analysis. A typical example is that of an operating system which, at boot time, carries out a set of initializations and then cyclically waits for, and produces, a variety of input/output operations. The states that belong to the ultimate cyclic behavioural component determine the central function of this type of system. The markings modeling such states are the home markings.

The existence of home markings is a widely studied subject in the theory of Petri nets [6, 1, 15, 2, 14, 4, 13], but only for very particular classes of them. Thus, in [1] it has been proven that live and 1-safe free-choice Petri nets have home markings. The result has successively been extended to live and safe free-choice Petri nets [15], live and safe equal-conflict Petri nets [14], and deterministically synchronized sequential process systems [11]. All these results make use, more or less directly, of a *confluence property* which is induced by liveness and safety.

The *home marking problem* for Petri nets (that is, the problem of deciding whether or not a given marking of a Petri net is a home marking) has been proven decidable in [5]. In our paper we show that this problem is undecidable for inhibitor Petri nets (section 2).

Then, we relate the concept of a home marking to the properties of confluence, safety, and noetherianity, and prove that confluent and noetherian Petri nets have an unique home marking (section 3). In Section 4 we define some versions of the state space inclusion problem for Petri nets, related to the home marking problem, and discuss their decidability status. We close the paper by some conclusions.

The rest of this section is devoted to a short introduction to Petri nets (for details the reader is referred to [12, 9]). A (finite) *Petri net* (with infinite capacities), abbreviated *PN*, is a 4-tuple $\Sigma = (S, T, F, W)$, where S and T are two finite non-empty sets (of *places* and *transitions*, respectively), $S \cap T = \emptyset$, $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation*, and $W : (S \times T) \cup (T \times S) \rightarrow \mathbf{N}$ is the *weight function* of Σ satisfying $W(x, y) = 0$ iff $(x, y) \notin F$. When all weights are one, Σ is called *ordinary*.

A *marking* of a Petri net Σ is a function $M : S \rightarrow \mathbf{N}$. A *marked Petri net*, abbreviated *mPN*, is a pair $\gamma = (\Sigma, M_0)$, where Σ is a *PN* and M_0 , the *initial marking* of γ , is a marking of Σ .

The behaviour of the net γ is given by the so-called *transition rule*, which consists of:

- (a) the *enabling rule*: a transition t is *enabled* at a marking M (in γ), abbreviated $M[t]_\gamma$, iff $W(s, t) \leq M(s)$, for any place s ;
- (b) the *computing rule*: if $M[t]_\gamma$ then t may *occur* yielding a new marking M' , abbreviated $M[t]_\gamma M'$, defined by $M'(s) = M(s) - W(s, t) + W(t, s)$, for any $s \in S$.

The transition rule is extended homomorphically to sequences of transitions by $M[\lambda]_\gamma M'$, and $M[wt]_\gamma M'$ whenever there is a marking M'' such that $M[w]_\gamma M''$ and $M''[t]_\gamma M'$, where M and M' are markings of γ , $w \in T^*$ and $t \in T$.

Let $\gamma = (\Sigma, M_0)$ be a marked Petri net. A word $w \in T^*$ is called a *transition sequence* of γ if there exists a marking M of γ such that $M_0[w]_\gamma M$. Moreover, the marking M is called *reachable* in γ . The set of all reachable markings of γ is denoted by $[M_0]_\gamma$ (or $[M_0]$ when γ is clear from context).

A Petri net γ is called *n-safe*, where $n \geq 1$ is a natural number, if $M(s) \leq n$ for all reachable markings M ; γ is called *safe* if it is *n-safe* for some n . Clearly, a Petri net is safe iff it has a finite set of reachable markings.

2 The Home Marking Problem

A *home marking* of a system is a marking which is reachable from every reachable marking in the system. For Petri nets, home markings are defined as follows.

Definition 2.1 A marking M of a Petri net $\gamma = (\Sigma, M_0)$ is called a *home marking* of γ if $M \in [M']$ for all $M' \in [M_0]$.

The Home Marking Problem (HMP)

Instance: $\gamma = (\Sigma, M_0)$ and a marking \overline{M} of γ ;

Question: is \overline{M} a home marking of γ ?

In [5], *home spaces* of Petri nets are considered. A home space of a Petri net γ is any set HS of markings of γ such that for any reachable marking M there is a marking $M' \in HS$ reachable from M . If HS is singleton, its unique element is a home marking.

A set A of markings of a Petri net γ is called *linear* if there are a marking M of γ and a finite set $\{M_1, \dots, M_n\}$ of markings of γ such that

$$(\forall M' \in A)(\forall 1 \leq i \leq n)(\exists k_i \in \mathbf{N})(M' = M + \sum_{i=1}^n k_i M_i).$$

The main result proved in [5] states that it is decidable whether or not a linear set of markings is a home space. Therefore, the home marking problem is decidable because any singleton set is linear.

The concept of a home marking can also be considered for extended Petri nets (like inhibitor, reset etc.) by taking into consideration their transition relation. In what follows we show that it is undecidable whether or not a marking of an inhibitor Petri net is a home marking. First, recall the concepts of an inhibitor net and counter machine.

A *k-inhibitor net* ($k \geq 0$) is a couple $\gamma = (\Sigma, I)$, where Σ is a net and I is a subset of $S \times T$ such that $F \cap I = \emptyset$ and $|\{s \in S \mid (s, t) \in I\}| \leq k$ for all $t \in T$.

Let $\gamma = (\Sigma, I)$ be an inhibitor net, M a marking of γ and $t \in T$. Then,

$$M[t]_{\gamma, i} \Leftrightarrow M[t]_{\Sigma} \wedge (\forall s \in S)((s, t) \in I \Rightarrow M(s) = 0),$$

and

$$M[t]_{\gamma, i} M' \Leftrightarrow M[t]_{\gamma, i} \wedge M[t]_{\Sigma} M'.$$

A *deterministic counter machine* (DCM) is a 6-tuple $A = (Q, q_0, q_f, C, x_0, I)$, where:

- (1) Q is a finite non-empty set of *states*, $q_0 \in Q$ is the *initial state*, and $q_f \in Q$ is the *final state*;
- (2) C is a finite non-empty set of *counters*. Each counter can store any natural number, and $x_0 : C \rightarrow \mathbf{N}$ is the initial content of the counters;
- (3) I is a finite set of *instructions*. For each state there is exactly an instruction that can be executed in that state; for q_f there is no instruction. An instruction for a state q is of the one of the following forms:

- *increment instruction* - $I(q, c, q')$

q : begin
 $c := c + 1$;
 go to q'
 end.

- *test instruction* - $I(q, c, q', q'')$
 q : if $c = 0$ then go to q'
 else begin
 $c := c - 1$;
 go to q''
 end.

Let $A = (Q, q_0, q_f, C, x_0, I)$ be a *DCM*. A *configuration* of A is a pair (q, x) , where $q \in Q$ and $x : C \rightarrow \mathbb{N}$. A configuration (q, x) is called *initial* when $q = q_0$ and $x = x_0$; a configuration (q, x) is called *final* when $q = q_f$.

Let $A = (Q, q_0, q_f, C, x_0, I)$ be a *DCM*. Define the binary relation \vdash_A on the configurations of A by:

$(q, x) \vdash_A (q', x')$ iff one of the following holds:

- (1) there is an increment instruction $I(q, c, q')$ such that $x'(c) = x(c) + 1$ and $x'(c') = x(c')$, $\forall c' \in C - \{c\}$;
- (2) there is a test instruction $I(q, c, q_1, q_2)$ such that
 - (2.1) if $x(c) = 0$, then $q' = q_1$ and $x' = x$;
 - (2.2) if $x(c) \neq 0$, then $q' = q_2$, $x'(c) = x(c) - 1$ and $x'(c') = x(c')$ for all $c' \in C - \{c\}$.

The Halting Problem for counter machines is to decide whether or not a given *DCM* reaches a final configuration. It is well-known that this problem is undecidable [10].

Theorem 2.1 The home marking problem for 1-inhibitor Petri nets is undecidable.

Proof We show that the halting problem for *DCM* can be reduced to the home marking problem for 1-inhibitor Petri nets.

Let $A = (Q, q_0, q_f, C, x_0, I)$ be a *DCM*. Define an 1-inhibitor Petri net as follows:

- to each $u \in Q \cup C$ we associate a place s_u ;
- to each increment instruction $I(q, c, q')$ we associate a transition t as in Figure 1(a), and to each test instruction $I(q, c, q', q'')$ we associate two transitions t' and t'' as in Figure 1(b).

A configuration $\sigma = (q, x)$ of A is simulated by the marking M given by:

$$\begin{aligned} M_\sigma(s_q) &= 1, \\ M_\sigma(s_{q'}) &= 0, \quad \forall q' \in Q - \{q\}, \\ M_\sigma(s_c) &= x(c), \quad \forall c \in C. \end{aligned}$$

Let M_0 be the marking corresponding to the initial configuration, and J be the set of pairs (s_c, t') , where s_c and t' are as in Figure 1(b).

The net $\gamma = (\Sigma, J, M_0)$ is an 1-inhibitor net, and we have:

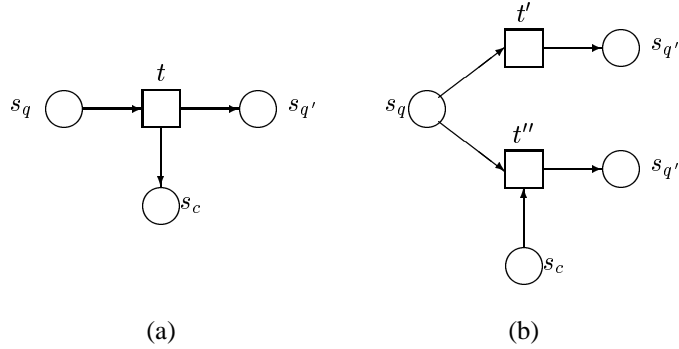


Figure 1: (a) The case $I(q, c, q')$; (b) The case $I(q, c, q', q'')$

(*) $\sigma = (q, x)$ is reachable in A from $\sigma_0 = (q_0, x_0)$ iff M_σ is reachable in γ from M_0 .

Modify now the net γ as in Figure 2 (all places and transitions of γ are pictorially represented in the dashed box labelled by γ ; the place s^* and the other transitions are new and specific to γ_1).

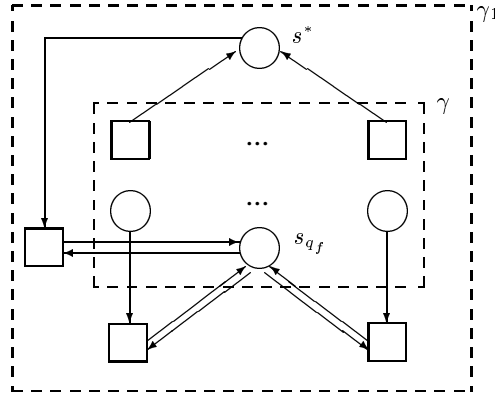


Figure 2: An inhibitor net instance associated to a DCM instance

We prove that A halts iff γ_1 has a home marking. Assume first that A halts, and let (q_f, x) be the final configuration when A halts. Then, $M_{(q_f, x)}(s_{q_f}) = 1$. Therefore, the newly added transitions can be applied yielding the marking $(1, 0, \dots, 0)$ which is a home marking of γ_1 (this marking can be reached from any reachable marking of γ_1 via the marking $M_{(q_f, x)}$).

Conversely, assume that γ_1 has home markings but A does not halt. Let M be a home marking of γ_1 . Then, $M(s_{q_f}) = 0$ (otherwise, A halts). Now we can easily see that the place s^* will be arbitrarily marked (each transition in A induces a transition in γ_1 which increases by one the place s^*) without the possibility to remove tokens from it be-

cause $M(s_{q_f}) = 0$. Therefore, M can not be reached from all reachable markings of γ_1 , contradicting the fact that M is a home marking of γ_1 . \square

3 Confluent and Noetherian Petri Nets

A Petri net is *confluent* if its firing relation is confluent, i.e., for any two reachable markings there is a marking reachable from both of them. This concept proved to be of great importance when we are dealing with the set of reachable markings of a Petri net. It has been considered explicitly for the first time, in connection with Petri nets, in [1], where it has been called *directedness*.

Definition 3.1 An mPN $\gamma = (\Sigma, M_0)$ is *confluent* if $[M_1] \cap [M_2] \neq \emptyset$ for all markings $M_1, M_2 \in [M_0]$.

Directly from definitions we obtain the following result.

Theorem 3.1 If an mPN has a home marking then it is confluent.

The converse of Theorem 3.1 does not hold generally. For example, the Petri net in Figure 3 is confluent but it does not have any home marking. In case of safe Petri nets, the

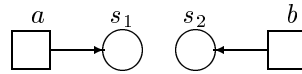


Figure 3: A confluent net which does not have a home marking

confluence property implies the existence of home markings.

Theorem 3.2 A safe mPN has a home marking iff it is confluent.

The proof of Theorem 3.2 is identical to the proof of Lemma 8.3 in [4] for ordinary Petri nets.

The concept of a *noetherian relation* is another very important concept in the theory of binary relations. As for the confluence property, a Petri net is called *noetherian* if its firing relation is noetherian.

Definition 3.2 An mPN is called *noetherian* if it does not have infinite transition sequences.

Theorem 3.3 Any confluent and noetherian marked Petri net has an unique home marking.

Proof Let $\gamma = (\Sigma, M_0)$ be a confluent and noetherian mPN . Since γ is noetherian, there is a marking $M' \in [M_0]$ such that $\neg(M'[t])$, for any transition t . We will show that M' is the unique home state of γ .

For every reachable marking M of γ the confluence property leads to the existence of a marking M'' such that $M'' \in [M] \cap [M']$. Then, the property of M' leads to the fact that $M'' = M'$. Therefore, $M' \in [M]$ which shows that M' is the unique home marking of γ . \square

Using the coverability tree of a Petri net [12, 9] we can easily prove that the noetherianity property is decidable.

Theorem 3.4 It is decidable whether an mPN is noetherian or not.

Proof An mPN γ is noetherian iff for any leaf node v of the coverability tree of γ , the label of v has no other occurrence on the path from the root to v . Since the coverability tree of a Petri net is always finite and can effectively be constructed, the property of being noetherian is decidable. \square

Let us denote by $\mathcal{C}(\mathcal{N}, \mathcal{H}, \mathcal{H}^*, \mathcal{S})$ the class of confluent (noetherian, having home markings, having an unique home marking, safe). It is easily seen that any noetherian mPN has a finite set of reachable markings (equivalently, it is a safe net). The converse of this statement does not hold generally as we can easily see from the net in Figure 4(a). A

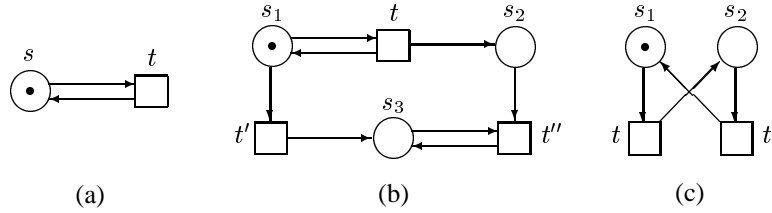


Figure 4: (a) $\gamma \in \mathcal{S} \cap \mathcal{H}^* - \mathcal{N}$; (b) $\gamma \in \mathcal{H}^* - \mathcal{S}$; (c) $\gamma \in \mathcal{S} \cap \mathcal{H} - \mathcal{H}^*$

pictorial view of the relationships between these classes of nets can be found in Figure 5. Some strict inclusions follow from the examples in Figure 4, and some of them are rather trivial.

It is important to know which nets are confluent. In [1] it has been proved that live and 1-safe free-choice Petri nets are confluent. The result has been extended in [15] to live and safe free-choice Petri nets. Further, Recalde and Silva proved in [14] that live and safe equal-conflict Petri nets have home markings (therefore, they are confluent), and the result has been extended to deterministically synchronized sequential process systems in [11].

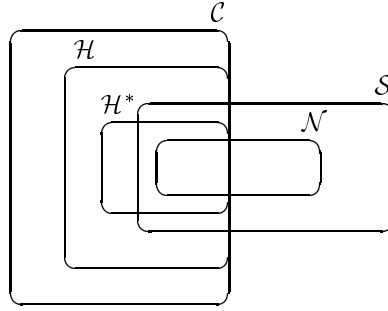


Figure 5: Relationships between classes of Petri nets

4 Home Markings and State Space Inclusions

The home marking problem can be naturally related to some particular versions of the space inclusion problem for Petri nets [7]. In order to define them we need first the following concept.

Definition 4.1 Let $\gamma = (\Sigma, M_0)$ be a *mPN* and \overline{M} a marking of γ . The *dual of γ w.r.t. \overline{M}* , denoted by $\overline{\gamma}$, is the Petri net defined as follows:

- $\overline{\gamma} = (\overline{\Sigma}, \overline{M})$;
- $\overline{\Sigma} = (S, \overline{T}, \overline{F}, \overline{W})$;
- $\overline{T} = \{\overline{t} | t \in T\}$;
- $(s, \overline{t}) \in \overline{F}$ iff $(t, s) \in F$, for all $s \in S$ and $t \in T$, and
 $(\overline{t}, s) \in \overline{F}$ iff $(s, t) \in F$, for all $s \in S$ and $t \in T$;
- $\overline{W}(s, \overline{t}) = W(t, s)$ and $\overline{W}(\overline{t}, s) = W(s, t)$, for all $s \in S$ and $t \in T$.

For a sequence $u = t_1 \cdots t_n$ of transitions of a Petri net Σ denote by \overline{u} the sequence $\overline{u} = \overline{t}_n \cdots \overline{t}_1$.

Lemma 4.1 Let Σ be a Petri net and M_1 and M_2 markings of Σ . Then, the following hold:

- (1) for every transition sequence $u \in T^*$, $M_1[u]_{\Sigma} M_2$ iff $M_2[\overline{u}]_{\overline{\Sigma}} M_1$;
- (2) M_2 is reachable from M_1 in Σ iff M_1 is reachable from M_2 in $\overline{\Sigma}$.

Proof (1) can be obtained by induction on the length of u using the fact that \overline{t} undoes the effect of t , and (2) follows from (1). \square

Now, we can prove the following simple but important result.

Proposition 4.1 Let $\gamma = (\Sigma, M_0)$ be a Petri net and \overline{M} a marking of γ . Then, \overline{M} is a home marking of γ iff $[M_0]_\gamma \subseteq [\overline{M}]_{\overline{\gamma}}$.

Proof Let us suppose first that \overline{M} is a home marking of γ . Then, for every marking $M \in [M_0]_\gamma$ there is a sequence of transitions $v \in T^*$ such that $M[v]_\gamma \overline{M}$. From Lemma 4.1 it follows that $\overline{M}[\overline{v}]_{\overline{\gamma}} M$, which shows that M is reachable from \overline{M} in $\overline{\gamma}$. Therefore, $[M_0]_\gamma \subseteq [\overline{M}]_{\overline{\gamma}}$.

Conversely, let M be a reachable marking in γ . The proposition's hypothesis leads to the fact that M is reachable in $\overline{\gamma}$. Then, from Lemma 4.1 it follows that \overline{M} is reachable from M in γ . Therefore, \overline{M} is a home marking of γ . \square

Recall now the space and sub-space inclusion problems as defined in [7] (in what follows, the components of the Petri net Σ_i will be denoted by S_i, T_i, F_i , and W_i , respectively).

The Space Inclusion Problem (SIP)

Instance: $\gamma_1 = (\Sigma_1, M_0^1)$ and $\gamma_2 = (\Sigma_2, M_0^2)$ such that $S_1 = S_2$;
 Question: does $[M_0^1]_{\gamma_1} \subseteq [M_0^2]_{\gamma_2}$ hold ?

The Sub-space Inclusion Problem (SSIP)

Instance: $\gamma_1 = (\Sigma_1, M_0^1)$, $\gamma_2 = (\Sigma_2, M_0^2)$, and $S \subseteq S_1 \cap S_2$;
 Question: does $[M_0^1]_{\gamma_1}|_S \subseteq [M_0^2]_{\gamma_2}|_S$ hold ?

It is known that both SIP and SSIP are undecidable [7]. Proposition 4.1 leads us to considering the following versions of SIP and SSIP (in what follows $\overline{\gamma}$ is the dual of γ w.r.t. a marking \overline{M} of γ).

The Dual Space Inclusion Problem (DSIP)

Instance: $\gamma = (\Sigma, M_0)$ and a marking \overline{M} of γ ;
 Question: does $[M_0]_\gamma \subseteq [\overline{M}]_{\overline{\gamma}}$ hold ?

The Dual Sub-space Inclusion Problem (DSSIP)

Instance: $\gamma = (\Sigma, M_0)$, a marking \overline{M} of γ , and $S' \subseteq S$;
 Question: does $[M_0]_\gamma|_{S'} \subseteq [\overline{M}]_{\overline{\gamma}}|_{S'}$ hold ?

From Proposition 4.1 it follows that HMP and DSIP are recursively equivalent and, therefore, DSIP is decidable because HMP is decidable [5].

Definition 4.2 A marking \overline{M} of a Petri net $\gamma = (\Sigma, M_0)$ is called a *home sub-marking* of γ w.r.t. $S' \subseteq S$ if for any marking $M \in [M_0]$ there is a marking $M' \in [M]$ such that $M'|_{S'} = \overline{M}|_{S'}$.

The Home Sub-marking Problem (HSMP)

Instance: $\gamma = (\Sigma, M_0)$, a marking \overline{M} of γ , and $S' \subseteq S$;
 Question: is \overline{M} a home sub-marking of γ w.r.t. S' ?

Our concept of a home sub-marking is, in fact, the same as that in [5] where it has been proven that the HSMP is decidable. HSMP and DSSIP are not recursively equivalent as HMP and DSIP are. In fact, we shall prove that DSSIP is undecidable for a proper subclass of Petri nets and, therefore, undecidable for the whole class of Petri nets.

Definition 4.3 A 3-tuple (Σ, s_1, s_2) is called a *two-way Petri net* (*2wPN*, for short) if Σ is a Petri net, s_1 and s_2 are places of Σ , and there is a partition of T , $T = T' \cup T''$, such that $\bullet s_1 = T' = s_1^\bullet$, $\bullet s_2 = T'' = s_2^\bullet$, and $W(s_1, t') = W(t', s_1) = W(s_2, t'') = W(t'', s_2) = 1$ for all $t' \in T'$ and $t'' \in T''$.

Pictorially, a *2wPN* is like in Figure 6 (its set of places is $S \cup \{s_1, s_2\}$, where $s_1 \neq s_2$ and $s_1, s_2 \notin S$).

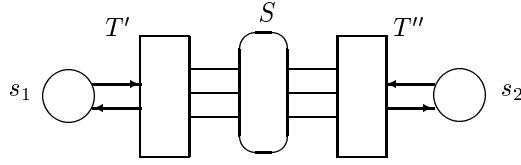


Figure 6: A pictorial view of a two-way Petri net

Theorem 4.1 The dual sub-space inclusion problem for *2wPN* is undecidable.

Proof We prove the undecidability of DSSIP by reducing SIP to it.

Let γ_1 and γ_2 be an instance of SIP. We consider the *2wPN* Σ as given in Figure 6, but with the following differences:

- $S = S_1 = S_2$;
- $T' = T_1$ and $T'' = \overline{T}_2$;
- the arcs and their weights between T_1 and S are given by F_1 and W_1 , respectively;
- the arcs and their weights between \overline{T}_2 and S are given by \overline{F}_2 and \overline{W}_2 , respectively.

Consider then the markings $M_0 = (M_0^1, 1, 0)$ and $\overline{M} = (M_0^2, 0, 1)$, and the marked Petri nets $\gamma = (\Sigma, M_0)$ and $\overline{\gamma} = (\Sigma, \overline{M})$.

Thus, we have obtained an instance of DSSIP for *2wPN* satisfying:

$$[M_0^1]_{\gamma_1} \subseteq [M_0^2]_{\gamma_2} \Leftrightarrow [M_0]_{\gamma}|_S \subseteq [\overline{M}]_{\overline{\gamma}}|_S.$$

Therefore, SIP is reducible to DSSIP for $2wPN$; the theorem follows then from the undecidability of SIP [7]. \square

Clearly, DSSIP for the whole class of Petri nets is undecidable, being undecidable for a sub-class of them.

Conclusions

The existence of home markings is a widely studied subject in the theory of Petri nets [6, 1, 15, 2, 14, 4, 13], but only for very particular classes of them. Thus, in [1] it has been proven that live and 1-safe free-choice Petri nets have home markings. The result has successively been extended to live and safe free-choice Petri nets [15], live and safe equal-conflict Petri nets [14], and deterministically synchronized sequential process systems [11]. All these results make use, more or less directly, of a confluence property which is induced by liveness and safety.

In this paper we have studied the home marking problem for Petri nets. We have proven several results that can be summarized as follows:

- the home marking problem for inhibitor Petri nets is undecidable;
- confluent and noetherian Petri nets have an unique home marking;
- the dual sub-space inclusion problem for Petri nets is undecidable.

All these results have been obtained by relating the concept of a home marking to some important concepts in Petri net theory, like confluence, noetherianity, and state space inclusion. Further study of these concepts is, in our opinion, an important subject of research.

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References

- [1] E. Best, K. Voss. *Free Choice Systems Have Home States*, Acta Informatica 21, 89–100, 1984.
- [2] E. Best, L. Cherkasova, J. Desel, J. Esparza. *Characterisation of Home States in Free Choice Systems*, Technical Report 9, Institut für Informatik, Universität Hildesheim, 1990.
- [3] R.V. Book, F. Otto. *String Rewriting Systems*, Springer-Verlag, 1993.
- [4] J. Desel, J. Esparza. *Free Choice Petri Nets*, Cambridge University Press, 1995.
- [5] D. Frutos-Escrig, C. Johnen. *Decidability of Home Space Property*, Technical Report LRI 503, 1989.

- [6] M. Hack. *Analysis of Production Schemata by Petri Nets*, M.S. Thesis, Project MAC TR-94, Massachusetts Institute of Technology, 1972 (Corrections in Computation Structures Note 17, 1974).
- [7] M. Hack. *Decidability Question for Petri Nets*, Technical Report 161, Laboratory for Computer Science, Massachusetts Institute of Technology, 1976.
- [8] M. Jantzen. *Confluent String Rewriting*, Springer-Verlag, 1988.
- [9] T. Jucan, F.L. Țiplea. *Petri Nets. Theory and Application*, Romanian Academy Publishing House, 1999.
- [10] M. Minsky. *Recursive Unsolvability of Post's Problem of TAG and other Topics in Theory of Turing Machines*, Annals of Mathematics, vol. 74, no. 3, 1961.
- [11] L. Recalde, E. Teruel, M. Silva. *Modeling and Analysis of Sequential Processes that Cooperate through Buffers*, IEEE Trans. Robotics Automat. 14(2). 1998, 267-277.
- [12] W. Reisig. *Petri Nets. An Introduction*, Springer-Verlag, 1985.
- [13] M. Silva, E. Teruel, J.M. Colom. *Linear Algebraic and Linear Programming Techniques for the Analysis of P/T Net Systems*, Lecture on Petri Nets I: Basic Models, Lecture Notes in Computer Science 1941, 1998, 309–373.
- [14] E. Teruel, M. Silva. *Liveness and Home States in Equal Conflict Systems*, Proc. of the 14th International Conference "Application and Theory of Petri Nets", Lecture Notes in Computer Science 691, 1993, 415–432.
- [15] W. Vogler. *Live and Bounded Free Choice Nets Have Home States*, Petri Net Newsletter 32, 1989, 18–21.