The Spatial Node Distribution of the Random Waypoint Mobility Model

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Abstract: The random waypoint model is a frequently used mobility model for simulation–based studies of wireless ad hoc networks. This paper investigates the spatial node distribution that results from using this model. We show and interpret simulation results on a square and circular system area, derive an analytical expression of the expected node distribution in one dimension, and give an approximation for the two–dimensional case. Finally, the concept of attraction areas and a modified random waypoint model, the random borderpoint model, is analyzed by simulation.

1 Introduction and Motivation

Ad hoc networks are self–organizing wireless communication systems that are formed by co–operating electronic devices (e.g., mobile computers, mobile phones, personal digital assistants, sensors). Such networks operate in a decentralized manner and do not rely on fixed network infrastructure. In general, all communication is wireless, and all stations may be mobile.

The modeling of the movement behavior of the stations is an important building block in simulation–based studies of mobile ad hoc networks. Mobility models are needed in the evaluation of protocols for medium access, power management, leader election, routing, and so on. The choice of the mobility model and its parameters has a significant influence on the obtained simulation results.

Researchers in this area can choose from a variety of models, which have been developed in the wireless communications and mobile computing community during the last decades (see e.g. [Gué87][MLTS97][ZD97][HGPC99][Bet01]). Also well–known motion models from physics and chemistry, such as random walks or Brownian motion, and models from other engineering disciplines, such as transportation theory [Hei89][BH00], are used in simulations of mobile ad hoc networks. Surveys and classifications on this topic can be found in [Bet01], [ZD97], and [LCW97].

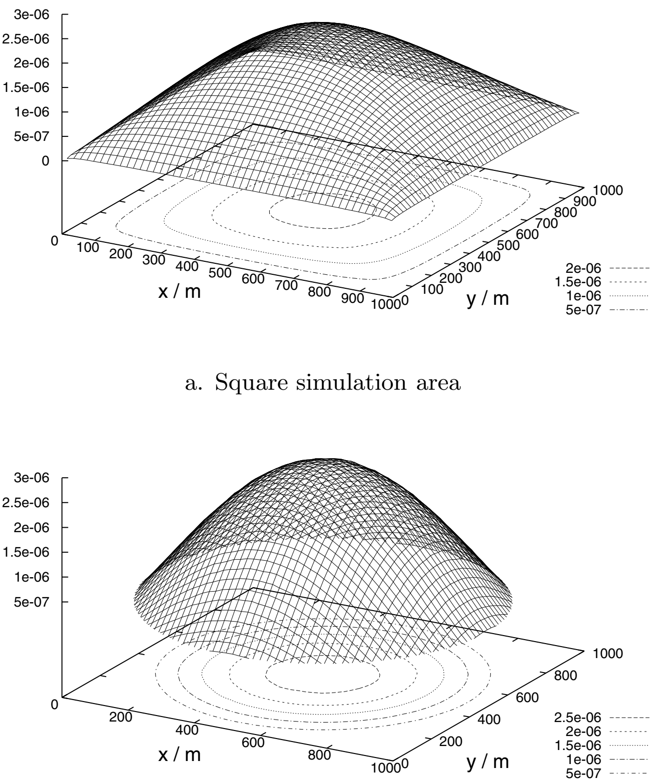
A very popular and frequently used mobility model in ad hoc networking research is the *random waypoint model* (see e.g. [BMJ+98][RP99][DPR00][HV99]). It is a simple and straightforward stochastic model that describes the movement behavior of a mobile network node in a two–dimensional system area as follows: A node randomly chooses a destination point in the area and moves with constant speed to this point. After waiting a certain pause time, it chooses a new destination, moves to this destination, and so on.

In a previous paper [Bet01], we noted that it is important to realize that the choice of the mobility model determines the resulting spatial node distribution during simulation. Although the initial positioning of the nodes is typically taken from a uniform distribution, the mobility model may change this distribution during simulation. This behavior usually occurs if the simulation area has borders. If we are not aware of how the used mobility model changes the node distribution, simulation results may be misinterpreted. This paper continues our research on this topic.

Our contributions are as follows: In Section 2, we analyze the random waypoint model in a typical simulation environment. The resulting spatial node distribution is shown for a square and a circular simulation plane. We explain why this particular, non–uniform distribution results and discuss the parameters influencing its shape. In Section 3, we outline the disadvantages and pitfalls of inhomogeneous node distributions in typical simulations of ad hoc and cellular networks. Section 4 derives an analytical equation for the node distribution of the random waypoint model in one dimension and verifies this formula by simulation. These results are then employed in Section 5 to give an analytical approximation of the distribution in two dimensions. The concept of "attraction areas" is considered in Section 6; and a modified random waypoint model, in which the destination points are only located at the borders of the area, is investigated in Section 7. Section 8 concludes this paper and outlines topics for further research.

2 Simulation–Based Study of the Node Distribution

In order to evaluate the spatial distribution of mobile stations that move according to the random waypoint model, we perform the following simulation: A node is randomly positioned on a $1000 \times 1000\,\mathrm{m}^2$ system area. A uniform random generator [MN98] chooses the x and y coordinates of a destination point; the node moves with constant speed v to this destination, randomly chooses a new destination (uniformly distributed), and so on. The time that the node takes to move from a starting position to its next destination is denoted as one *movement epoch*. Without loss of generality, we set the pause time in the destination point to zero.



a. Square simulation area

b. Circular simulation area (disc)

Figure 1: Spatial node distribution resulting from the random waypoint mobility model: Simulation results

As the node moves around during simulation, we always trace its current position. To do so, we divide the entire area into square cells of size 20×20 m^2. Each cell is represented in a two–dimensional 50×50 histogram $h(\xi, \psi; t)$, with $\xi,\ \psi\ \in \{0, 1, \ldots 49\}$ and simulation time t. For each movement epoch, the duration of how long the node stays in a particular cell (the "dwell time") is added to the respective field in the histogram. Since the node moves with constant speed, the dwell time is directly proportional to the distance that the node covers in this cell. The simulation ends after $20 \cdot 10^6$ movement epochs, which yields an acceptable confidence interval of the results. Finally, the histogram is normalized with the total movement time and the size of the cells, such that its volume is 1. Figure 1a shows the resulting normalized distribution $\overline{h}(x, y)$ and contour lines for certain occurrence values. After elimination of transient effects at the beginning of the simulation, the distribution remains stable for long simulation times. We make the following observations and discuss them:

The distribution has a peak in the middle of the area, i.e., a node is most likely to be found in the central cells of the simulation area ($x,\ y \in [480\,\mathrm{m},\ 520\,\mathrm{m}]$). The probability that a node is located at the border of the area goes to zero. Furthermore, the distribution is symmetric in all four axis directions from the center.

The reason for this inhomogeneous distribution is obvious: In order to set the direction of a node, the random waypoint model chooses a uniformly distributed destination point $(x_d,\ y_d)$, rather than a uniformly distributed angle $\varphi \in [0, 2\pi[$. Therefore, nodes located at the border of the simulation area are very likely to move back toward the middle of the area. For example, a node located at $(x,\ y) = (100\ \mathrm{m},\ 100\ \mathrm{m})$ chooses with much higher probability a new destination point in the direction $\varphi \in]0,\ \frac{\pi}{2}[$ than a point toward a border or the edge, i.e., $\varphi \in]\frac{\pi}{2},\ 2\pi[$. Most likely, it chooses a destination point that requires the node to pass the middle of the simulation area; in this example, $\varphi = \frac{\pi}{4}$.

Let us regard the inner contour line of Fig. 1a. Its shape can be approximated by a circle of radius 130 m around the middle. From the value of the contour line ($2 \cdot 10^{-6}$) we can state that a node is located more than $2 \cdot 10^{-6} \cdot 130^2\pi = 11\%$ of its simulation time within this disc, while the disc covers only about 5% of the entire simulation area.

A simulation with a higher node speed v yields the same normalized histogram $\overline{h}(x, y)$; the shape of the distribution only depends on the size and shape of the simulation area and the distribution of the destination points.

We repeat our experiment in a circular simulation area of radius $r_m = 500$ m around the point $(x,\ y) = (500\ \mathrm{m},\ 500\ \mathrm{m})$. The resulting distribution is shown in Figure 1b. A similar qualitative behavior as in the square case can be observed. In addition, the distribution is now circular symmetrical, i.e., the occurrence of a node only depends on the distance r from the center and not on its position angle ϕ, i.e., $\overline{h}(r, \phi) = \overline{h}(r)$, with the polar coordinates $r = \sqrt{(x - 500\,\mathrm{m})^2 + (y - 500\,\mathrm{m})^2}$ and $\phi = \arctan(\frac{y - 500\,\mathrm{m}}{x - 500\,\mathrm{m}})$. (Note that the current polar coordinate ϕ of a node and its current movement direction φ are different parameters.)

Regarding the second contour line from the maximum (at a distance of $r_0 = 250\mathrm{m}$), we can say that a node is located at least 39% of its simulation time within this disc ($2 \cdot 10^{-6} \cdot r_0^2\pi = 0.39$), although the disc covers only 25% of the total area.

3 Consequences of Inhomogeneous Node Distributions

Let us now discuss why these observations are of particular importance. Before we consider mobile ad hoc networks, let us imagine that we evaluate the call blocking and call dropping rates in an F/TDMA cellular network (see, e.g., [Lin97]). Such a simulation can be performed, for example, to plan the radio resources of a GSM network. We assume that the entire simulation area is divided into cells of equal size, and each cell has been assigned an equal and fixed (or quasi–static) number of channels. If a new call of a user does not obtain a channel, the call will be *blocked*. If an ongoing call of a mobile user that changes its cell cannot be handed over to the new cell (because there is no channel available), the call must be *dropped*. If we use the random waypoint model in this simulation–based study, we will (in the mean) always achieve a higher call blocking and dropping rate in the center cells, because here the user density is the highest. A similar situation occurs in an evaluation of dynamic channel assignment algorithms: We will need (in the mean) a higher number of channels in the middle of the area. If we are not aware that this phenomenon is a consequence of the mobility model, we may misinterpret the results and draw wrong conclusions.

Also in simulations of mobile ad hoc networks, an inhomogeneous node distribution is often not convenient and may create a pitfall. For example, if more nodes are located in the middle, these nodes have on average a higher "connectivity degree" than nodes at the border, even if a toroidal distance metric [BK01] is used that avoids other border effects. This has a major consequence, for example, in evaluations of distributed power control algorithms. Most important, however, is that several theoretical and analytical investigations of ad hoc networks assume a random uniform distribution of the nodes (see, e.g., work on the capacity of ad hoc networks [GK00a][GK00b]). Results obtained in simulations that use the random waypoint model cannot be compared (or proven) with these investigations.

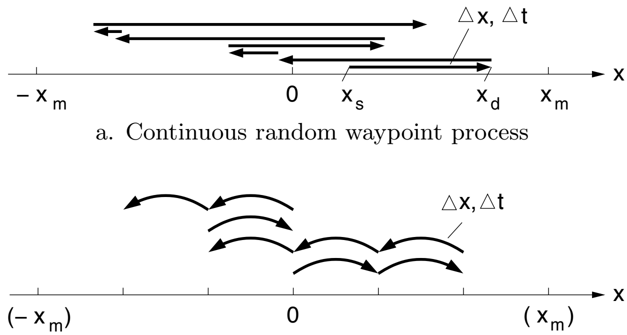
Our observations *do not mean* that the random waypoint model is inappropriate for simulations; maybe it even models movement effects and user distributions of real life better than other models do. However, it *does mean* that researchers must be aware of the node distribution of this model, in order to draw correct conclusions from simulation results.

4 Analytical Derivation of the Distribution in 1 Dimension

Having seen the spatial node distribution from a simulation, our aim is now to *calculate* this distribution for a given system area. We first consider a one–dimensional random waypoint movement as illustrated in Figure 2a.

4.1 Problem Statement

At time $t = 0$, a node is positioned at a location x_s on a finite line $[-x_m, x_m]$. It randomly chooses a destination point x_d on this line and moves with constant speed v to this point. From this point, it chooses a new destination point x_d, and so on. As in the two–dimensional case, the time from a starting position x_s to the next destination x_d is denoted as one movement epoch. The length of a given epoch i is $\Delta x(i) = |x_s(i) - x_d(i)|$. The speed v of the node remains constant during



a. Continuous random waypoint process

b. Discrete random walk process

Figure 2: One–dimensional random movement processes

the entire movement procedure (not just within one movement epoch), and there is no pause time in the destination points. Therefore, the duration of an epoch i is $\Delta t(i) = \frac{\Delta x(i)}{v}$. Typically, the destination points are taken from a uniform distribution, i.e.,

$$f_{X_d}(x_d) = \begin{cases} \frac{1}{2x_m} & \text{for } -x_m \leq x_d \leq x_m \\ 0 & \text{else} \end{cases} . \tag{1}$$

Since a destination point of the current movement epoch is the starting point x_s for the next epoch, the starting points are also uniformly distributed between $-x_m$ and x_m, i.e.,

$$f_{X_s}(x_s) = \begin{cases} \frac{1}{2x_m} & \text{for } -x_m \leq x_s \leq x_m \\ 0 & \text{else} \end{cases} . \tag{2}$$

This movement model can be described as a stochastic process. Let the continuous random variable $X(t)$ denote the location of a node, which moves according to the random waypoint model, at a given time t. The value range of X is given by $X \in [-x_m,\, x_m]$.

In the following, our goal is to derive the *probability density function (PDF)* $f_X(x)$, assuming that the movement process continues for a very long time $t \to \infty$. With a given $f_X(x)$, we can then calculate the probability that a node is located between x_a and x_b, where $x_a < x_b$, by

$$P\left(x_a \leq X \leq x_b\right) = \int_{x_a}^{x_b} f_X(x)\, dx. \tag{3}$$

The probability that a node is located within a very small interval $\pm\delta x$ around a point x_0, i.e. within $[x_0 - \delta x, x_0 + \delta x]$, is

$$P\left(x_0 - \delta x \leq X \leq x_0 + \delta x\right) = \int_{x_0-\delta x}^{x_0+\delta x} f_X(x)\, dx \;\approx\; 2\,\delta x\, f_X(x_0). \tag{4}$$

4.2 Comparison with Random Walk and Brownian Motion

Let us first compare the random waypoint process with the well–known family of *random walk* processes. A *simple random walk* [Ros96][BW90] is a discrete stochastic process, defined as follows (see Fig. 2b): In each movement epoch, a node must decide to jump a distance Δx to the right or left side. The random variable X, i.e., the location of a node, can therefore only take discrete values $X = \pm k\Delta x$, $k \in Z$. A node chooses to move to the right side with probability p and to the left side with probability $1 - p$. The current direction decision is independent of the previous one.

As opposed to the random waypoint movement, such a random walk assumes an infinite value range for X, i.e., the node moves unrestrictedly on an infinite line. But there also exists the theory of *bounded random walks*, in which a node moves on a finite line with fixed borders. When it reaches the border, it can be reflected [Pap84][Law95] or absorbed (i.e., it stays there forever) [Law95]. Also a repositioning at the center [Bur01] or a "wrap around" border behavior, which is equivalent to a movement process on a circle, is sometimes useful. For each of these border behaviors, a discrete probability density function $f_X(x)$ of the node location X at time t and several other stochastic properties can be calculated (see, e.g., [TSG01]).

The most obvious difference between the random walk and random waypoint process is as follows: A random walk is a location and time–discrete process, and each movement epoch has the same length Δx and the same duration Δt. The random waypoint movement, however, is a location and time–continuous process, in which the epoch lengths and durations vary from epoch to epoch (i.e., $\Delta x(i)$, $\Delta t(i)$). Besides this, there are several other properties of the random waypoint model that raise difficulties for its analyzation.

For example, if we try to model the random waypoint movement as a discrete stochastic process (by dividing the continuous value range of X into equidistant intervals), a node would "jump" from x_s to x_d. This would ignore all locations x between x_s and x_d that are traversed by a node that follows a random waypoint movement (see Fig. 2). Furthermore, in a discrete random waypoint model, the probabilities of going right or left, p and $1-p$, would have to depend on the current position $X(t)$ of the node, i.e., the current direction decision is correlated to the decision of the previous movement epoch. For example, a node located close to the right border of the line will most probably choose a destination point at its left hand side (uniform distribution of destinations x_d). Also the expected change in position $\Delta x(i)$ of a certain epoch i depends on the starting point $x_s(i)$.

So–called *continuous–time random walks* [PB00][MS84] just model a pause time after each movement epoch (the pause time has a continuous probability distribution) — which does not provide a solution to our problem.

The limiting form of a random walk with $\Delta t \to 0$, which yields a continuous–state process, is called *Wiener process* or *Brownian motion* [Law95][Pap84]. As in our random waypoint process, $X(t)$ is here a continuous function of t, i.e., there are no "jumps" in the path $X(t)$ over t. Many statistic properties for both unlimited and bounded Brownian motion can be derived (see, e.g., [Pap84][Law95][Ros96]).

Nevertheless, we are currently not in the position to generalize and map the well–known stochastic characteristics of Brownian motion and/or random walks to our random waypoint process or to find a mathematical process that is equal to the random direction process. Let us therefore take our own approach to derive the PDF $f_X(x)$.

4.3 Derivation of the Probability Density Function $f_X(x)$

We note that in a continuous movement process, *probability* can be interpreted in terms of *time*: The time that a node can be found in the interval $[x_a, x_b]$ during the process, divided by the total running time of the process t, follows Equation (3) as the process runs very long. Another basic observation is that in a movement process with constant speed, the *time* and *distance* that a node covers during its movement are directly proportional to each other (see Fig. 2a). In other words, longer movement epochs contribute with a "higher weight" to the PDF of X.

We therefore consider the following approach: At each time instant t, the current position $X(t)$ of a random waypoint node is traced and added to a one–dimensional, continuous histogram $h(x;\, t)$. The histogram states how often a point x has been visited by the node. We normalize this histogram, such that the integral over all values is 1, i.e.

$$\overline{h}(x;\, t) = \frac{h(x;\, t)}{\int_{-\infty}^{\infty} h(x;\, t)\, dx}\,, \tag{5}$$

and record it for a very long simulation time. Since the normalized distribution $\overline{h}(x;t)$ converges if the simulation time is long enough, we can skip the time index t and define $\overline{h}(x) := \overline{h}(x, t \to \infty)$. This yields the wanted PDF

$$f_X(x) = \lim_{t\to\infty} \overline{h}(x;\, t). \tag{6}$$

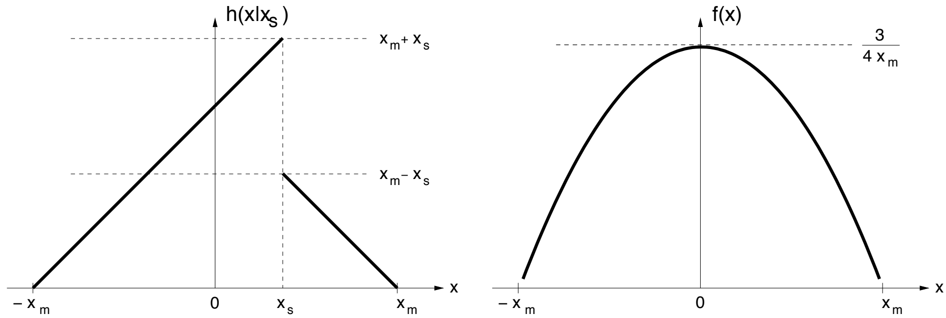
One movement epoch of a node, say between time t_1 and t_2, from a starting point $X(t_1) = x_s$ to a destination point $X(t_2) = x_d$, increases the histogram $h(x,\, t_1)$ by a value of $+1$ for x between x_s and x_d. Using the Heaviside unit step function

$$u(x - x_0) = \begin{cases} 1 & \text{for } x > x_0 \\ 0 & \text{for } x < x_0 \end{cases}\,, \tag{7}$$

we can write

$$h(x,\, t_2) \;=\; h(x,\, t_1) + \left| u\left(x - X(t_1)\right) - u\left(x - X(t_2)\right) \right|. \tag{8}$$

Note that in a random walk, only the endpoints x_s and x_d would contribute to the histogram. Since we do not know the starting and ending points $(x_s(i),\, x_d(i))$



a. Function $h(x \mid x_s)$ for given source point x_s and uniformly distr. dest. points x_d

b. PDF $f_X(x)$ of a node's location X

Figure 3: Derivation of node distribution of a one–dim. random waypoint movement

of the movement epochs in the movement process, we must evaluate them as an average on all possible points, being weighted with their distributions $f_{X_s}(x_s)$ and $f_{X_d}(x_d)$. (In the following, we skip the indices X, X_s, and X_d.) For uniformly distributed starting and destinations points we put up

$$\begin{aligned} f(x) &= \frac{1}{C_1} \iint_{-\infty}^{\infty} |u(x-x_s) - u(x-x_d)| \cdot \; f(x_d)\, f(x_s)\, dx_s\, dx_d \\ &= \frac{1}{4x_m^2\, C_1} \iint_{-x_m}^{x_m} |u(x-x_s) - u(x-x_d)| \; dx_s\, dx_d\ . \end{aligned} \tag{9}$$

We first integrate over all possible destination points x_d for a given, fixed starting point x_d, and denote this function as $h(x \mid x_s)$:

$$\begin{aligned} h(x \mid x_s) &= \int_{-x_m}^{x_m} |u(x-x_s) - u(x-x_d)| \; dx_d \\ &= \begin{cases} x + x_m & \text{for } -x_m \le x < x_s \\ -x + x_m & \text{for } x_s < x \le x_m \\ 0 & \text{else} \end{cases} \\ &= (x + x_m)\ \left(u(x+x_m) - u(x-x_s) \right) \\ &+ \ (-x + x_m)\ \left(u(x-x_s) - u(x-x_m) \right). \end{aligned} \tag{10}$$

This function is illustrated in Figure 3a. The value at $x = x_s$ is not defined. Note that if we normalized $h(x \mid x_s)$, we would obtain the PDF of a process in which a node is positioned at a known starting point x_s, chooses a destination point x_d from a uniform distribution, and moves to this point with constant speed. It is then positioned again at x_s, randomly chooses a destination point, and so on.

But let us continue with the calculation of $f(x)$. With (9) and (10) we have

$$f(x) \quad = \quad \frac{1}{4x_m^2\, C_1} \int_{-x_m}^{x_m} h(x \mid x_s)\, dx_s \tag{11}$$

for $-x_m \le x \le x_m$, and 0 otherwise. The integral is $\int_{-x_m}^{x_m} h(x|x_s)\, dx_s = -2\left(x^2 - x_m^2\right)$ for $-x_m \le x \le x_m$. Since $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$ must hold, and $-2 \cdot \int_{-x_m}^{x_m} \left(x^2 - x_m^2\right) dx = \frac{8}{3}\, x_m^3$, we obtain the normalization term $C_1 = \frac{4}{3} x_m^2$, and can conclude with the following theorem.

Theorem 1: A node moves on a line $[-x_m, x_m]$ according to a one–dimensional random waypoint model with constant speed and uniformly distributed destination points. The probability density function $f_X(x)$ of its location X is given by

$$\boxed{f_X(x) = -\frac{3}{4x_m^3}\, x^2 + \frac{3}{4x_m} \quad , \text{for} - x_m \le x \le x_m} \; . \tag{12}$$

This function is illustrated in Figure 3b. The probability that a node can be found at the borders goes to zero, i.e.,

$$P(x_m \mp \delta x \le X \le \pm x_m) = \lim_{x \to \pm x_m} \delta x\, f_X(x) = 0\,. \tag{13}$$

The expected value of X is

$$E\{X\} = \int_{-\infty}^{\infty} x\, f(x)\, dx = 0\,, \tag{14}$$

and the probability that a node is not more than Δx away from $x = 0$ is

$$P(-\Delta x \le X \le \Delta x) = -\frac{1}{2}\left(\frac{\Delta x}{x_m}\right)^3 + \frac{3}{2}\left(\frac{\Delta x}{x_m}\right). \tag{15}$$

For example, a node resides 68.75% of its movement time within $[-\frac{x_m}{2}, +\frac{x_m}{2}]$, i.e., within the inner half of the line.

Let us verify these results by a simulation. Figure 4 shows the normalized histogram $\overline{h}(x)$ from a simulation with 10^6 movement epochs and the PDF $f(x)$. The analytical curve goes exactly through the simulation points.

Sometimes it is more convenient to regard a random waypoint movement on a line $[0,\, x_m]$, rather than on $[-x_m,\, x_m]$. We can easily state the following corollary.

Corollary 1: The density function of a node's location that performs a random waypoint movement on a line $[0, x_m]$, with constant speed and uniformly distributed destination points, is

$$\boxed{f_X(x) = -\frac{6}{x_m^3}\, x^2 + \frac{6}{x_m^2}\, x \;, \text{for } 0 \le x \le x_m} \; . \tag{16}$$

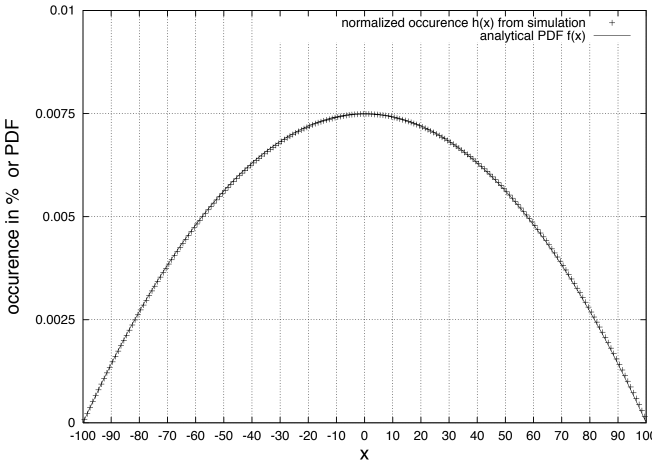


Figure 4: Spatial node distribution resulting from a 1D random waypoint mobility model: Normalized histogram $\overline{h}(x)$ from simulation and PDF $f(x)$

5 Approximation of the Distribution in 2 Dimensions

We now consider the two–dimensional case again and give approximations for the PDFs in a square and circular system area, i.e., $f(x,\, y)$ and $f(r,\, \phi) = f(r)$.

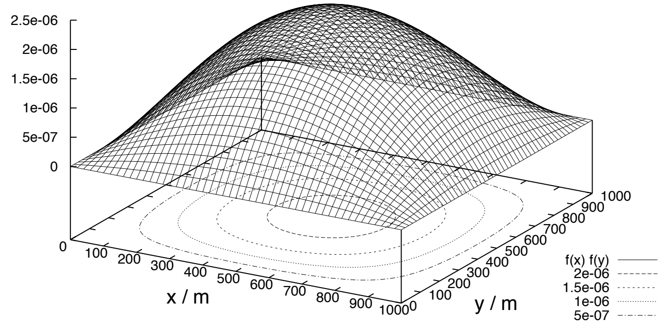
5.1 Square Area

Let us *assume* that the two–dimensional movement in a square area consists of two independent one–dimensional movement processes along the x and y axes as described in the previous section. The plot of the function

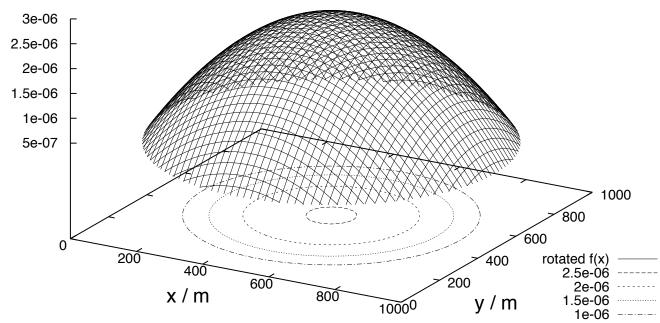
$$f(x)\, f(y) = \frac{9}{16\, x_m^3\, y_m^3} \left(x^2 - x_m^2\right) \left(y^2 - y_m^2\right), \tag{17}$$

shown in Fig. 5a (offset $x = y = 500$ m) looks similar to the actual distribution $\overline{h}(x,\, y)$ of Fig. 1a. We observe that the shapes of the two curves almost match in an area around the center, whereas there is a difference in the border regions. There is a difference between the two curves, because the projection of a two–dimensional movement does not have a constant speed. Thus, the projection of $\overline{h}(x,\, y)$ onto one axis is not exactly $f(x)$ as calculated in the one–dimensional case. However, we can state that $f(x, y) \approx f(x)\, f(y)$.

The "expected point" of $f(x) f(y)$ is $E\left\{(x,\, y)\right\} = (0, 0)$. The same point represents the maximum with a value $\frac{9}{16\, x_m\, y_m}$. For our example $x_m = y_m = 500$ m, we have a value of $2.25 \cdot 10^{-6}$.



a. Square simulation area: $f(x)\, f(y)$



b. Circular simulation area (disc): $f(x)$ rotated

Figure 5: Analytical approximation of the spatial node distribution of the waypoint mobility model

5.2 Circular Area

Now we consider a circular area of radius r_m. We substitute $x \to r$ in (12), rotate it around the maximum, and normalize the resulting function, such that $\int_0^{r_m} \int_0^{2\pi} f(r,\, \phi)\, r\, dr\, d\phi = 1$. This yields

$$f(r, \phi) = f(r) = -\frac{2}{r_m^4\, \pi}\, r^2 + \frac{2}{r_m^2\, \pi} \tag{18}$$

for $0 \leq r \leq r_m$. Its maximum value is $f(r = 0) = \frac{2}{r_m^2\, \pi}$, which is $2.55 \cdot 10^{-6}$ for $r_m = 500$ m. A plot of this function, with $r_m = 500$ m and an offset $(x,\, y) = (500\,\text{m}, 500\,\text{m})$ is shown in Figure 5b. If we compare this curve with the simulation results of Figure 1b, we can conclude that (18) is a good approximation for the exact spatial node distribution $f(r, \phi)$ on a disc.

The probability that a given mobile node is located within a circle of radius $r = r_0$ from the center can be approximated by

$$P(r \leq r_0) = \int_0^{r_0} \int_0^{2\pi} f(r,\, \phi)\, r\, dr\, d\phi = \frac{r_0^2 \left(2r_m^2 - r_0^2\right)}{r_m^4}\,, \tag{19}$$

for $0 \leq r_0 \leq r_m$. Clearly, $P(r \leq r_m) = 1$. For a simulation area of radius $r_m = 500$ m and $r_0 = 250$ m (the same example as in Section 2), we obtain $P(r \leq 250\,\text{m}) = 44\%$. In other words, a node is about 44% of its total movement time in this disc.

6 Hot Spots and Attraction Areas

After these theoretical investigations, let us come back to a simulation–based analysis. Sometimes it is desired to model a *hot spot* in a simulation, i.e., a certain area in which many nodes are located. Since the resulting node distribution of the random waypoint model (Fig. 1) is independent of the starting values of the nodes (for long simulation times), we cannot create such a hot spot by placing many nodes in a certain area at the beginning of the simulation, as is possible with other mobility models. Actually, the random waypoint model automatically creates a hot spot in the middle of the simulation area.

However, we can easily create an *attraction area* anywhere in the simulation area by using an inhomogeneous distribution of the destination points. When randomly selecting a destination point, a node chooses a point in this area with a higher probability than a point outside this area. In a rectangular system area, with $-x_m \leq x \leq x_m$ and $-y_m \leq y \leq y_m$ and an attraction area defined by $x_{a,min} \leq x \leq x_{a,max}$ and $y_{a,min} \leq y \leq y_{a,max}$, the distribution of the destination points $(x_d,\, y_d)$ could be

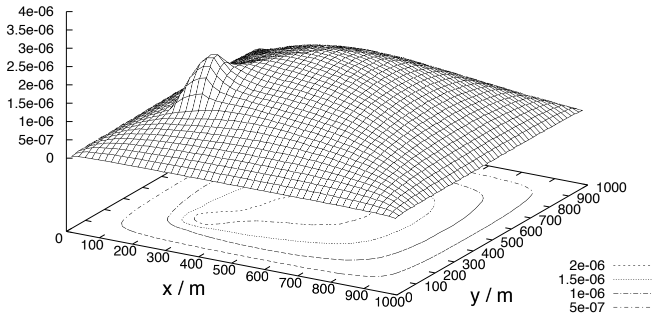
$$\begin{aligned} f(x_d,\, y_d) \;=\; & \tfrac{1}{A+(\mu-1)A_a} \Big[\big(u(x_d + x_m) - u(x_d - x_m) \big) \big(u(y_d + y_m) \\ & - u(y_d - y_m) \big) + (\mu - 1)\big(u(x_d - x_{a,max}) - u(x_d - x_{a,min}) \big) \\ & \big(u(y_d - y_{a,max}) - u(y_d - y_{a,min}) \big) \Big] \end{aligned} \tag{20}$$

where μ is the *intensity* of the attraction area. With the sizes of the areas $A = 2x_m + 2y_m$ and $A_a = (x_{a,max} - x_{a,min})(y_{a,max} - y_{a,min})$, the normalization term is $A + (\mu - 1)A_a$. Fig. 6 gives three examples for attraction areas of different size and intensity.

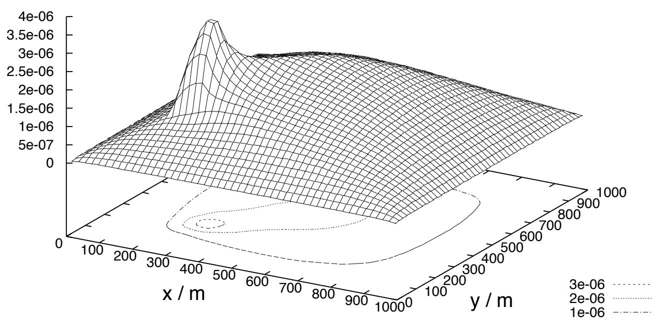
7 A Modified Random Waypoint Model

Our last investigation considers a modified random waypoint model, in which the chosen destination points can only be located at the borders of the system area. For example, the destination points in a circular area are taken from

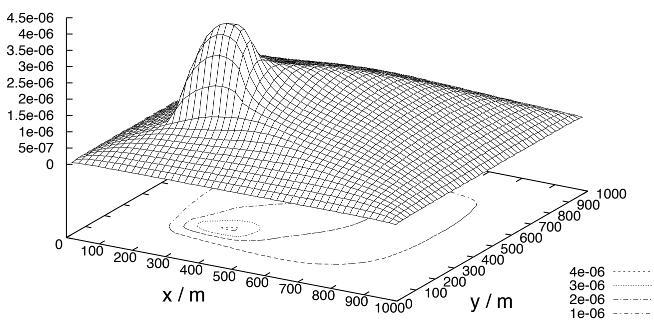
$$f(r_d,\, \phi_d) = \begin{cases} \frac{1}{2\pi r_m} & \text{for } r_d = r_m,\; 0 \leq \phi_d < 2\pi \\ 0 & \text{else} \end{cases} \tag{21}$$



a. Attraction area [200, 300][200, 300] with intensity $\mu = 5$

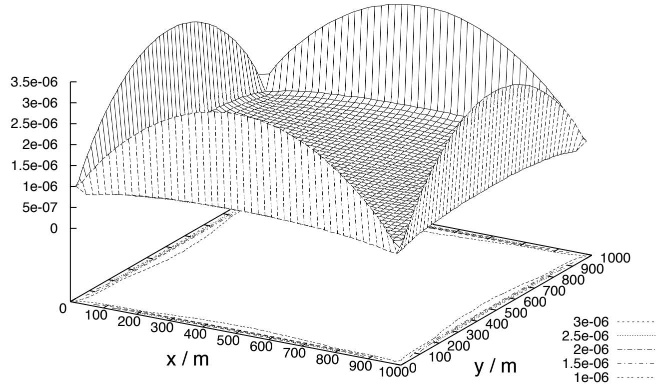


b. Attraction area [200, 300][200, 300] with intensity $\mu = 10$

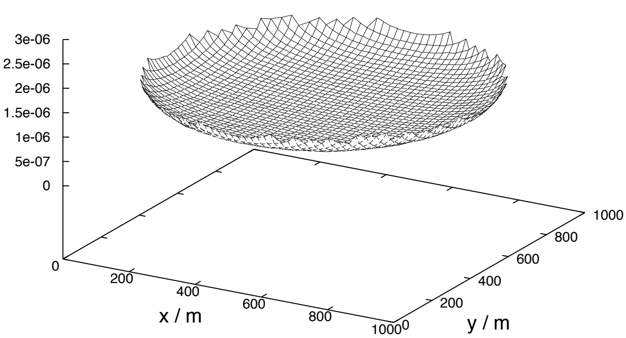


c. Attraction area [200, 400][200, 300] with intensity $\mu = 10$

Figure 6: Node distribution of a random waypoint model with attraction areas



a. Square simulation area



b. Circular simulation area (disc)

Figure 7: Spatial node distribution resulting from the random borderpoint mobility model: Simulation results

with $r_d = \sqrt{x_d^2 + y_d^2}$ and $\phi_d = \arctan(\frac{y_d}{x_d})$. We denote this model as *random borderpoint model*.

As illustrated in Fig. 7a, such a model yields a rather unrealistic node distribution when being applied in a rectangular area. In this case, a node located at the border chooses a destination point on the same border with high probability and then moves along this border line.

The random borderpoint model on a circular area achieves an interesting result. As shown in Fig. 7b, the nodes are almost uniformly distributed now (compared with Fig. 1b). Such a simple modification could be used to overcome the disadvantages of the inhomogeneous node distribution described above.

8 Conclusions

In this paper, we discussed the inhomogeneous spatial node distribution resulting from the well–known random waypoint mobility model. We first illustrated the distribution by simulation in a two–dimensional square and a disc (Fig. 1). Next, we derived an analytical expression for a one–dimensional random waypoint movement (Section 4, Equ. (12)) and gave approximations for the two dimensional case (Section 5). Finally, we considered the node distribution with attraction areas of different intensity (Section 6) and investigated a modified random waypoint model, denoted here as random borderpoint model (Section 7). Applying the latter on a disc yields a smoothed node distribution, which is much closer to a uniform distribution.

In further research, one could derive the exact PDFs for the two–dimensional case and investigate the effects of inhomogeneous node distributions on various kinds of performance simulations of ad hoc and cellular networks.

An alternative to the random waypoint model is a simple *random direction model* in which a new direction $\varphi \in [0 \ldots 2\pi[$ is chosen after a random movement epoch time, rather than a destination point. It has approximately the same complexity and programming effort. In this model, nodes have always a uniformly distributed angle within $[0 \ldots 2\pi[$. They can also cross the borders of the simulation area and should then be bounced back or "wrapped around" to the other side of the area, which results in a uniform node distribution. Such a model should be preferred, if a homogeneous distribution is desired, for example to compare simulation results with analytical investigation (which are often derived for uniform node placements).

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