Ranking Specific Sets of Objects

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Abstract: Ranking sets of objects based on an order between the single elements has been thoroughly studied in the literature. In particular, it has been shown that it is in general impossible to find a total ranking – jointly satisfying properties as dominance and independence – on the whole power set of objects. However, in many formalisms from the area of knowledge representation one does not need to order the entire power set, since certain sets are already ruled out due to hard constraints or are not satisfying some background theory. In this paper, we address the question whether an order on a given subset of the power set of elements satisfying different variants of dominance and independence can be found. We first show that this problem is tractable when we look for partial rankings, but becomes NP-complete for total rankings.

Keywords: Ranking Sets. Complexity.

1 Introduction

The problem of lifting rankings on objects to ranking on sets has been studied from many different view points – see [BBP04] for an excellent survey. Several properties (also called axioms) have been proposed in order to indicate whether the lifted ranking reflects the given order on the elements, among them dominance and independence. Roughly speaking, dominance ensures that adding an element which is better (worse) than all elements in a set, makes the augmented set better (worse) than the original one. Independence, on the other hand, states that adding an element a to sets A and B where A is already known to be preferred over B, must not make $B \cup \{a\}$ be preferred over $A \cup \{a\}$ (or, in the strict variant, $A \cup \{a\}$ should remain preferred over $B \cup \{a\}$). It is well known that constructing a ranking on the whole power set of objects which jointly satisfies such properties is in general not possible.

However, in many situations one does not need to order the entire power set. Just take preference-based formalisms from the area of knowledge representation. In fact, there exists a wide variety of approaches that proceed in the following way: in a first step, a set S of models that satisfy some hard constraints is obtained; then the most preferred models out of S are identified from an order on S that stems from some simpler principles, for instance, from an order of atomic elements. Such formalisms come in different veins. Qualitative Choice Logic [BBB04], for instance, adds a dedicated connective to standard propositional logic to express preferences between formulas; other formalisms like Answer-Set Optimization Problems [BNT03] or Qualitative Optimization Problems [FTW13] separate the specification of hard constraints and soft constraints. However, in both cases an implicit

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order between models is constructed that is used to deliver the preferred models. The relation to the problem of ranking sets is evident. Atoms in the specification play the role of elements; models (containing elements that are set to true) of the theories can be seen as the sets to be ranked. Typically these formalisms allow a more general specification of the order than just ranking the atoms. However, the latter can be done in these formalisms as well. To the best of our knowledge, there are no investigations whether the implicit ordering of models satisfies principles like dominance or independence. More generally, the problem of lifting rankings to such specific sets of elements seems to be rather neglected, so far. The only exception we are aware of deals with subsets of a fixed cardinality [Bo95].

This indicates that is worth studying under which conditions rankings of elements can be lifted to arbitrary subsets of the power set of elements such that desirable properties hold. In order to do so, we first give a new definition for dominance which appears more suitable in such a setting. Then, we consider the following problem: Given a ranking on elements, and a set S of sets of elements, does there exist a strict (partial) order on S that satisfies D and I (where D is either standard dominance or our notion of dominance and I is independence or strict independence)? We show that the problem is either trivial or easy to solve for the case of strict partial orders. Our main result is NP-completeness for the case when strict orders are considered.

The remainder of the paper is organised as follows. In the next section, we recall some basic concepts. In Section 3 we discuss why standard dominance can be seen as too weak in our setting and propose an alternative definition. Section 4 contains our main results. Due to the space restriction, proofs are omitted.

2 Background

The formal framework we want to consider in the following consists of a finite[3], nonempty set X, equipped with a linear order $<$ and a subset $\mathscr{X} \subseteq \mathscr{P}(X) \setminus \{\emptyset\}$ of the power set of X not containing the empty set. We want to find an order $\prec$ on $\mathscr{X}$, that satisfies some niceness conditions. We will consider several kinds of orders. We recall the relevant definitions.

Definition 1. A binary relation is called a *strict partial order*, if it is irreflexive and transitive. A *strict order* is a total strict partial order. A binary relation is called a *preorder*, if it is reflexive and transitive. A *weak order* is a total preorder. If $\preceq$ is a weak order on a set X, for all $x,y \in X$, the *corresponding strict order* $\prec$ is defined by $x \prec y \leftrightarrow (x \preceq y \wedge y \not\preceq x)$.

Definition 2. Let $A \in \mathscr{X}$ be a set of elements of X. Then we write $\max(A)$ for the (unique) element of A satisfying $\max(A) > a$ for all $a \in A \setminus \{\max(A)\}$. Analogously, we write $\min(A)$ for the (unique) element of A such that $\min(A) < a$ holds for all $a \in A \setminus \{\min(A)\}$. Furthermore, we say a relation R on a set $\mathscr{X}$ *extends* a relation S on $\mathscr{X}$ if xSy implies xRy for all $x,y \in \mathscr{X}$. Finally, we say a relation R on $\mathscr{X}$ is the *transitive closure* of a relation S

[3] In the literature, infinite sets of alternatives are also considered. The results presented in the background section hold also for this case. For the results on computational complexity in the main part, finiteness of the (infinitely many) instances is obviously crucial.

on $\mathscr{X}$ if the existence of a sequence $x_1Sx_2S\dots Sx_k$ implies x_1Rx_k for all $x_1, x_k \in \mathscr{X}$ and R is the smallest relation with this property. We write $trcl(S)$ for the transitive closure of S.

Many different axioms a good order should satisfy are discussed in the literature (an overview over the relevant interpretations and the corresponding axioms can be found in the survey [BBP04]). The following axioms "have very plausible intuitive interpretations" [BBP04, p.11] for decision making under complete uncertainty and belong to the most extensively studied axioms in the literature. We added conditions of the form $X \in \mathscr{X}$ that are not necessary if $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$ holds.

Axiom 1 (Extension Rule)**.** For all $x, y \in X$ if $\{x\}, \{y\} \in \mathscr{X}$ then,

$$\{x\} \prec \{y\} \text{ iff } x < y$$

Axiom 2 (Dominance)**.** For all $A \in \mathscr{X}$ and all $x \in X$ if $A \cup \{x\} \in \mathscr{X}$ then,

$$y < x \text{ for all } y \in A \text{ implies } A \prec A \cup \{x\}$$

$$x < y \text{ for all } y \in A \text{ implies } A \cup \{x\} \prec A$$

Axiom 3 (Independence)**.** For all $A, B \in \mathscr{X}$ and for all $x \in X \setminus (A \cup B)$, if $A \cup \{x\}, B \cup \{x\} \in \mathscr{X}$ then

$$A \prec B \text{ implies } A \cup \{x\} \preceq B \cup \{x\}$$

Axiom 4 (Strict Independence)**.** For all $A, B \in \mathscr{X}$ and for all $x \in X \setminus (A \cup B)$ if $A \cup \{x\}, B \cup \{x\} \in \mathscr{X}$ then

$$A \prec B \text{ implies } A \cup \{x\} \prec B \cup \{x\}$$

Every reasonable order should satisfy the extension rule. However, in the case $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$, the extension rule is implied by dominance [BBP04]. A natural task is to find an order on $\mathscr{P}(X) \setminus \{\emptyset\}$ that satisfies dominance together with (some version of) independence. However, in their seminal paper [KP84], Kannai and Peleg have shown that this is impossible for regular independence and dominance if $|X| \geq 6$ and $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$ hold. Barberà and Pattanaik [BP84] showed that for strict independence and dominance this is impossible even for $|X| \geq 3$ and $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$.

If we abandon the condition $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$, the situation is not as clear. There are, obviously, sets $\mathscr{X}$ such that there is an order on $\mathscr{X}$ satisfying strict independence and dominance – for example, $\mathscr{X} = \{A\}$ for some set $A \in \mathscr{P}(X) \setminus \{\emptyset\}$. Every order on $\mathscr{X}$ satisfies all axioms proposed in this section by trivial means.

3 Setting the Stage

Many useful results regularly used in the setting of $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$ are not true in the more general case. For example, in contrast to the result stated above, the extension rule is, in general, not implied by dominance.

Example 3. Consider the example $X = \{x_1, x_2\}$ with $x_1 < x_2$ and $\mathscr{X} = \{\{x_1\}, \{x_2\}\}$ with $\{x_2\} \prec \{x_1\}$. The extension rule is clearly violated in this example, however dominance is vacuously true as no set with two elements exists.

It could be argued that dominance should imply the extension rule. However the regular formulation of dominance is not strong enough in our setting because the intermediate set $\{x_1, x_2\}$ is missing from $\mathscr{X}$. In general, some natural consequences of dominance in the case of $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$ rely on the availability of intermediate sets. Therefore, it is reasonable to ask for stronger versions of dominance that behave nicely in the general case. We observe that $x < y$ for all $y \in A$ implies obviously $\max(A \cup \{x\}) = \max(A)$ and $\min(A \cup \{x\}) < \min(A)$; whereas $x > y$ for all $y \in A$ implies $\max(A \cup \{x\}) < \max(A)$ and $\min(A \cup \{x\}) = \min(A)$. We can use this property to define another notion of dominance.

Axiom 5 (Maximal Dominance)**.** For all $A, B \in \mathscr{X}$,

$$(\max(A) \leq \max(B) \wedge \min(A) < \min(B)) \vee$$
$$(\max(A) < \max(B) \wedge \min(A) \leq \min(B)) \rightarrow A \prec B$$

This axiom trivially implies the extension rule. Additionally, if $\mathscr{X}$ is sufficiently large, for example $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$, dominance and independence imply maximal dominance. It would be possible to define several other versions of dominance of intermediate strength. In this work we will make do with dominance and maximal dominance. We will see that the results justify this approach.

Furthermore, we also want to look at the case where additionally the requirement that $\prec$ has to be total is dropped. If we consider (maximal) dominance and strict independence we can look at partial orders to treat this problem. However, dominance talks about strict preferences while independence produces weak preferences. Therefore, neither a partial order, nor a preorder can satisfy dominance and independence for syntactical reasons. The natural solution, using a preorder and defining the corresponding partial order by $a < b$ iff $a \leq b$ but not $a \geq b$ holds, is not possible because we want to define orders recursively. Especially, we want to add weak and strict preferences independently. To solve this problem, we define a new type of order (or, more accurately, a pair of orders). This definition is supposed to emulate the interplay between an order and its corresponding strict order in partial and preorders.

Definition 4. A pair of relations $(\leq, <)$ on a set $\mathscr{X}$ is an *incomplete order*, if (1) $\leq$ is a preorder; (2) $<$ is a partial order; and (3) $x < y \leq z$ implies $x < z$ and $x \leq y < z$ implies $x < z$ for all $x, y, z \in \mathscr{X}$.

Every incomplete order is part of a order and its corresponding strict order.

Proposition 5. *For every incomplete order $(\leq_i, <_i)$, there is an order $\leq$ with a corresponding strict order $<$ such that $\leq$ extends $\leq_i$ and $<$ extends $<_i$.*

	Dom + Ind	Max Dom +Ind	Dom + Strict Ind	Max Dom + Strict Ind
Not total	always	always	in P	in P
Total	NP-c.	NP-c.	NP-c.	NP-c.

Tab. 1: Main results

4 Main Results

We studied in total 8 problems as defined below.[4] Our results are summarized in Table 1.

The Partial (Max)-Dominance-Strict-Independence Problem. *Given a linearly ordered set X and a set $\mathscr{X} \subseteq \mathscr{P}(X) \setminus \{\emptyset\}$, decide if there is a partial order on $\mathscr{X}$ satisfying (maximal) dominance and strict independence.*

The Partial (Max)-Dominance-Independence Problem. *Given a linearly ordered set X and a set $\mathscr{X} \subseteq \mathscr{P}(X) \setminus \{\emptyset\}$, decide if there is an incomplete order on $\mathscr{X}$ satisfying (maximal) dominance and independence.*

The (Max)-Dominance-(Strict)-Independence Problem. *Given a finite linearly ordered set X and a set $\mathscr{X} \subseteq \mathscr{P}(X) \setminus \{\emptyset\}$, is there a (strict) total order on $\mathscr{X}$ satisfying (maximal) dominance and (strict) independence.*

4.1 Partial Orders

First we want to classify sets that allow for a partial order satisfying dominance and strict independence. To do so, we build a minimal transitive relation satisfying dominance and strict independence. First, we build a minimal transitive relation satisfying dominance. It is worth noting that a very similar relation can be defined for maximal dominance. Using this relation, all results in this section can be proven for maximal dominance the same way.

Definition 6. The relation $\prec_d$ is defined as follows. For all $A, A \cup \{x\} \in \mathscr{X}$, (1) $A \prec_d A \cup \{x\}$ if $y < x$ for all $y \in A$; and (2) $A \cup \{x\} \prec_d A$ if $x < y$ for all $y \in A$. The relation $\prec_d^t$ is defined as $\prec_d^t := \mathrm{trcl}(\prec_d)$.

It is easy to see that $\prec_d^t$ is a partial order and that a partial order on a set $\mathscr{X}$ satisfies dominance if and only if it extends $\prec_d^t$. We want to extend this relation to a minimal relation for strict independence and dominance.

Definition 7. We build a relation $\prec_\infty$ by induction. We start with $\prec_0^t := \prec_d$. For $\prec_{n+1}$ we select sets $A, B, A \setminus \{x\}, B \setminus \{x\} \in \mathscr{X}$ with $x \in X$ and $A \setminus \{x\} \prec_n^t B \setminus \{x\}$ and set $A \prec_{n+1} B$. Then we define $\prec_{n+1}^t := trcl(\prec_{n+1})$ and $\prec_\infty = \bigcup_n \prec_n^t$.

[4] Definitions that only differ in one or two words are combined into one definition using brackets.

This relation satisfies dominance, strict independence and transitivity by construction. However, in general, $\prec_\infty$ is not irreflexive. If $\prec_\infty$ is not irreflexive no strict partial order can extend it. This rules out the existence of a partial order satisfying dominance and strict independence, as every strict partial order on $\mathscr{X}$ satisfying dominance and strict independence is an extension of $\prec_\infty$. Hence, if we want to decide if a set allows a partial order that satisfies strict independence and dominance we only have to check if $\prec_\infty$ is irreflexive on $\mathscr{X}$. This can clearly be done in polynomial time.

Theorem 8. *The partial Dominance-Strict-Independence problem is in P.*

Additionally, we can characterize the sets allowing such partial orders in a more combinatorial fashion using the following definition of links. We then show that links indeed characterize $\prec_\infty$.

Definition 9. A $\prec_\infty$*-link* from A to B in $\mathscr{X}$ is a sequence $A =: C_0, C_1, \ldots, C_n := B$ with $C_i \in \mathscr{X}$ for all $i \leq n$ such that, for all $i < n$, either $C_i \prec_d C_{i+1}$ holds or there is a link between $C_i \setminus \{x\}$ and $C_{i+1} \setminus \{x\}$ for some $x \in X$.

Lemma 10. *For $A, B \in \mathscr{X}$, $A \prec_\infty B$ holds if and only if there is a $\prec_\infty$-link from A to B.*

This gives us an easy characterization of sets $\mathscr{X}$ with irreflexive $\prec_\infty$

Corollary 11. *$\prec_\infty$ is irreflexive if and only if there is no set $A \in \mathscr{X}$ such that there is a $\prec_\infty$-link from A to A.*

We would like to produce a similar result for dominance and (non-strict) independence. As already discussed, we have to build an incomplete instead of partial order.

Definition 12. We build a pair of relations $(\preceq^\infty, \prec^\infty)$ by induction. We start with $\prec_t^0 := \prec_d$ and $A \preceq_t^0 A$ for all A. For $\preceq^{n+1}$ we select sets $A, B, A \setminus \{x\}, B \setminus \{x\} \in \mathscr{X}$ with $x \in X$ and $A \setminus \{x\} \prec_t^n B \setminus \{x\}$ and set $A \preceq^{n+1} B$. Then we set

$$\prec_t^{n+1} := trcl(\prec^{n+1}) \cup \big\{(A,B) \mid \exists C (A \prec^{n+1} C \preceq^{n+1} B \vee A \preceq^{n+1} C \prec^{n+1} B)\big\}$$

and $\preceq_t^{n+1} = trcl(\preceq_t^{n+1})$. Finally we set $\preceq^\infty = \bigcup_n \preceq_t^n$ and $\prec^\infty = \bigcup_n \prec_t^n$.

It is clear that this pair satisfies dominance and independence. Furthermore, it is obvious that $\prec_\infty$ and $\preceq_\infty$ are transitive, that $\preceq$ is reflexive and that the pair satisfies condition (3) of an incomplete order. Furthermore, it is easy to see that $A \preceq^\infty B$ implies $\max(A) \leq \max(B)$ and $\min(A) \leq \min(B)$ possibly without strict preference and $A \prec^\infty B$ implies $\max(A) \leq \max(B)$ and $\min(A) \leq \min(B)$ with at least one strict preference. This property clearly implies the irreflexivity of $\prec^\infty$. Therefore we have an incomplete order satisfying dominance and independence on all sets.

Theorem 13. *$(\preceq_\infty, \prec_\infty)$ is an incomplete order satisfying dominance and independence on all sets $\mathscr{X} \subseteq \mathscr{P}(X) \setminus \{\emptyset\}$.*

This includes the case $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$. Here, we even get maximal dominance for free.

Proposition 14. *Let $\mathscr{X} = \mathscr{P}(X) \setminus \{\emptyset\}$. Then $(\preceq^\infty, \prec^\infty)$ is the minimal incomplete order satisfying maximal dominance and independence.*

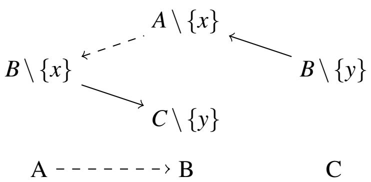

Fig. 1: Family that forces that $A \prec B$ leads to $B \prec C$

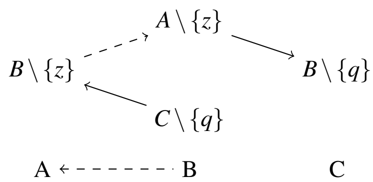

Fig. 2: Family that forces that $A \succ B$ leads to $B \succ C$

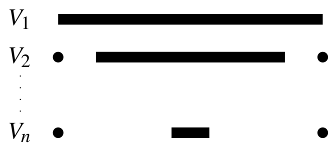

Fig. 3: Sketch of the sets V_1, V_2 and V_n

4.2 Total Orders

We show that it is, in general, not possible to construct a (strict) total order satisfying (maximal) dominance and (strict) independence deterministically in polynomial time. We do this by a reduction from betweenness.

Problem (Betweenness)**.** Given a finite set $V = \{v_1, v_2, \ldots, v_n\}$ and a set of triples $R \subseteq V^3$, find a strict total order on V such that $a < b < c$ or $a > b > c$ holds for all $(a,b,c) \in R$.

Betweenness is known to be NP-hard [Op79]. We use this result to show NP-hardness for all four versions of the of the (Max)-Dominance-(Strict)-Independence problem. We sketch the reduction for the maximal dominance and strict independence case.

We represent a triple (a,b,c) in a Max-Dominance-Strict-Independence instance with sets A,B,C and force $A \prec B \prec C$ or $A \succ B \succ C$ by adding two families of sets $A \setminus \{x\}, B \setminus \{x\}, B \setminus \{y\}, C \setminus \{y\}$ and $A \setminus \{z\}, B \setminus \{z\}, B \setminus \{q\}, C \setminus \{q\}$ where q,x,y,z are elements of X. Furthermore, we add sets that enforce $B \setminus \{x\} \prec C \setminus \{y\}$, $B \setminus \{y\} \prec A \setminus \{x\}$, $C \setminus \{q\} \prec B \setminus \{z\}$ and $A \setminus \{z\} \prec B \setminus \{q\}$ by dominance and strict independence. Figure 1 shows the case that $A \prec B$ holds for a strict total order $\prec$ on $\mathscr{X}$ satisfying maximal dominance and strict independence. We show that $B \prec C$ has to hold in this case. $A \setminus \{x\} \succ B \setminus \{x\}$ would contradict strict independence, therefore $A \setminus \{x\} \prec B \setminus \{x\}$ has to hold. Hence, $B \setminus \{y\} \prec C \setminus \{y\}$ has to hold because otherwise, $A \setminus \{x\} \prec B \setminus \{x\} \prec C \setminus \{y\} \prec B \setminus \{y\} \prec A \setminus \{x\}$ would be a circle contradicting the irreflexity of $\prec$. This implies $B \prec C$ by strict independence. Figure 2 shows, that $A \succ B$ leads to $B \succ C$ analogously.

We code the elements $v_1, \ldots, v_n$ of V for a betweenness instance (V,R) by sets $V_1, V_2, \ldots, V_n$ such that such that all sets have the same maximal and minimal element and the second largest elements are decreasing and second smallest elements are increasing (see Figure 3). Then we can enforce $V_i \prec V_j$ by adding $V_i \setminus \{\max(V_i)\}$ and $V_j \setminus \{\max(V_j)\}$ if $i < j$ holds or $V_i \setminus \{\min(V_i)\}$ and $V_j \setminus \{\min(V_j)\}$ if $i > j$ holds.

The actual construction goes like this: Let (V,R) be an instance of betweenness with $V = \{v_1,\ldots,v_n\}$. We construct an instance of the Max-Dominance-Strict-Independence problem. We set $X = \{1,2,\ldots,N\}$ equipped with the usual linear order, where $N = 8n^3+2n$. We construct $\mathscr{X}$ according to the idea discussed above. $\mathscr{X}$ contains sets representing $v_1,\ldots,v_n$ of the following form:

$$V_i := \{1,N\} \cup \{i+1,i+2,\ldots,N-i\}$$

Furthermore, for every triple we add the following sets: Pick a triple $(v_i,v_j,v_k) \in R$ and set $k = n+1+8i$ for the i-th triple. Let $(A,B,C) = (V_i,V_j,V_k)$ be the triple of sets coding the triple of elements (v_i,v_j,v_k). We add the following sets:

$$A \setminus \{k\}, B \setminus \{k\}, B \setminus \{k+1\}, C \setminus \{k+1\}, A \setminus \{k+2\}, B \setminus \{k+2\}, B \setminus \{k+3\}, C \setminus \{k+3\}$$

These sets correspond to the sets $A \setminus \{x\}, B \setminus \{x\}, \ldots, C \setminus \{q\}$ in Figure 1 and Figure 2. Observe that the inductive construction guarantees that every constructed set is unique. We now have to force the preferences $B \setminus \{k+1\} \prec A \setminus \{k\}$, $B \setminus \{k\} \prec C \setminus \{k+1\}$, $A \setminus \{k+2\} \prec B \setminus \{k+3\}$ and $C \setminus \{k+3\} \prec B \setminus \{k+2\}$.

Firstly, for technical reasons[5], we add sets $A \setminus \{k,k+4\}, B \setminus \{k+1,k+4\}$. Then, observe that, by construction, either $A \setminus \{1,k,k+4\} \succ B \setminus \{1,k+1,k+4\}$ or $A \setminus \{k,k+4,N\} \succ B \setminus \{k+1,k+4,N\}$ is implied by maximal dominance. We add $A \setminus \{k,k+4,1\}$ and $B \setminus \{k+1,k+4,1\}$ in the first case and $A \setminus \{k,k+4,N\}$ and $B \setminus \{k+1,k+4,N\}$ in the second case. This ensures $B \setminus \{k+1\} \prec A \setminus \{k\}$ by strict independence. In the same way, we can force the other preferences using $k+5, k+6$ and $k+7$ instead of $k+4$.

Finally, we pick a new triple $(v'_i,v'_j,v'_k) \in R$ until we treated all triples in R. Observe that there are at most n^3 triple, thus, for every triple, the values $k,\ldots,k+7$ lie between $n+1$ and $N-n$, hence are element of every V_i. In total, we add 24 sets per triple. Therefore, $\mathscr{X}$ contains $n+24n^3$ sets. It can be checked that (V,R) is a positive instance of betweenness if and only if $\mathscr{X}$ allows for a total strict order that satisfies maximal dominance and strict independence.

Theorem 15. *The Max-Dominance-Strict-Independence problem, the Max-Dominance-Independence problem, the Dominance-Strict-Independence problem and the Dominance-independence problem are NP-complete.*

5 Conclusion

We have shown that the problem of deciding whether a linear order can be lifted to a ranking of sets of objects satisfying a form of dominance axiom and a form of independence is in P or trivial if we do not expect the ranking to be total and NP-complete if we do. In order to prove P-membership or triviality we constructed rankings. These rankings could be used in applications e.g to eliminate obviously inferior sets of objects from a set of options. On the

[5] This makes sure that we don't accidentally force a preference between $A \setminus \{k\}$ and $B \setminus \{k\}$.

other hand, one could argue that the NP-hardness result shows that, in general, an ordering on objects does not give sufficient information to decide preferences between some sets of objects and therefore we are forced to guess preferences.

Future work concerns the complexity of the studied problem if the sets are given in a compact way, for instance in terms of a formula such that the models characterize the sets to be ranked. Another item on our agenda is to investigate whether the proposed logic in [GE11] can be used for specific sets of objects as well.

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