Utilization of Heterogeneity in Modular Reachability Analysis for Petri Nets

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A *modular Petri net* is composed of multiple individual Petri nets, the modules. Modules are composed by fusing their *interface* transitions. The behavior of *internal* transitions is unrelated to other modules and is recorded in *local reachability graphs* for each module.

Fusion vectors describe which interface transitions of which modules need to synchronize for their firing. This behavior is recorded in the *synchronization graph*, linking the local reachability graphs together. For this, internal behavior between two firings of interface transitions is abstracted to form *segments*. A vertex in the synchronization graph collects segments for each module and records the changes from synchronized interface transitions. The *modular state space* [1] consists of the local reachability graphs and the synchronization graph.

The modular state space can be used to analyze the reachability of markings in Petri nets. However, constructing the local reachability graphs for each module can be highly inefficient because of the state explosion problem [3]. In this paper, we show how the construction of the local reachability graph can be circumvented if the given modules have a certain topology. We study for two different classes of Petri nets how their structural properties can be used to construct a *reduced* modular state space. The two classes are acyclic Petri nets and state machines.

In modular Petri nets with acyclic modules, the state equation is utilized to bypass the construction of the local reachability graphs. This is because in acyclic Petri nets, the solvability of the state equation is not only necessary but also sufficient for the reachability of a marking [2]. Information about the local reachability graph can therefore be obtained by solving a linear system of equations (ILP). As a result, all reachable markings of a module can be derived from initial markings using linear algebra. In order to construct the reduced modular state space, the interacting behavior of the modules must still be represented. Accordingly, all reachable markings that activate interface transitions are required for each segment in order to correctly reflect the behavior in the synchronization graph. We do not fully explore the segments, as we can deduce all reachable markings that activate an interface transition with the help of the state equation. Processing segments in local reachability graphs of acyclic modules with ILPs still leads to the complete and correct modular state space.

We develop a method for modular Petri nets with state machine modules that avoids the construction of local reachability graphs. A *state machine* is a Petri net system, where every transition has exactly one pre- and one post place. These structural restrictions make it possible to decompose the set of places of the net. The decomposition is performed using the strongly connected

components (SCC), the equivalence classes of strongly connected places. Within a single SCC, every distribution of present tokens is possible; therefore, only the number of tokens per component is sufficient for further processing of a marking. The backward cones of the individual components determine the way in which the tokens can move within the net. This can be expressed as an ILP from a given initial marking. As a result, statements can be made about the local state space based solely on the distribution of the tokens of an initial marker without having to construct it. Therefore, only the initial markings and the activating markings for the interface transitions need to be stored for each segment in order to reflect the behavior of the modular Petri net. The modular state space is still represented correctly and completely by processing the segments using these ILPs. The special case of strongly connected state machines allows us to use only the number of tokens to represent a segment. This is because for modules of this structure, any marking that has the same token count as the initial marking is reachable.

We show how the reachability of markings can be verified in the reduced modular state space of Petri nets with some modules of these considered net classes.

References

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