Robust Likelihood Function for Blind Mobile Terminal Localization

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Abstract: The focus of this contribution is put on the blind localization of mobile terminals in urban scenarios. The proposed localization method exploits multipath propagation, which is typical of urban terrain. It is assumed that each multipath component is characterized by its direction of arrival (DoA) and relative time of arrival (RToA) parameter. In essence the developed method compares the multipath parameters predicted by the ray tracing algorithm with those measured by the single observing station. In this context inevitably occur a multipath order problem, which is tackled in detail within a proposed likelihood function definition. The latter follows an approach known from the robust statistics. Its intention is to provide a robust and reliable position estimate having at hand minimum a priori information about system parameters.

1 Introduction

There is a rapid growth of wireless applications that require the knowledge of the mobile terminal's location [KH06]. In most cases the cooperative position estimation methods [GG05] can be used. In this contribution, however, we concentrate on the *blind mobile localization* (BML), which presumes no cooperation of the mobile terminal referred to as *mobile station* (MS) with the *observing station* (OS). This problem is typical of non-subscribed user localization, e.g. in emergency, security, and safety applications [KST06].

In the preceding paper [ADKT08] we formulated the boundary conditions and proposed a possible solution of the BML task. Its goal is to develop a method for geo locating of a MS in a non-cooperative mode, i.e. solely from the signals radiated by the MS, implying that neither MS nor the cellular infrastructure are involved in the positioning process. The idea of the proposed BML method implies a correlation of measured direction of arrival (DoA) and relative time of arrival (RToA) parameters with data pre-calculated by the ray tracing (RT) analysis [Mau05]. Formulation of an appropriate correlation criterion is a key task of the underlying likelihood function and turns into a challenge in the noisy case. Since the measurement process induces missing detections of the true propagation paths or conversely produces the false multipaths. That is, the number of true multipaths as well as the corresponding multipath parameter are distorted, which makes the unique mapping impossible. In particular, the *multipath model order* problem, i.e. a need to compare hypothetical positions with different number of predicted multipaths, requires careful consideration. In [ADKT08] the probabilistically motivated likelihood function was proposed. It relies on the known error statistics, i.e. false alarm rate and detection probability. The likelihood definition presented in this work does not need a priori knowledge about clutter parameter, which is a more realistic assumption when working with real-life data. Furthermore, it is able to cope with the multipath model order problem.

The remainder of the paper is organized as follows. In section 2 we present the measurement error model and in section 3 the robust version of the likelihood function for the BML is introduced.

2 Measurement model

We start with introduction of frequently used symbols. Let $\boldsymbol{\xi}_{\mathrm{O}}$ denote the known OS position. The MS position $\boldsymbol{\xi}_{\mathrm{M}}$ is unknown and must be estimated. For this purpose we introduce some hypothetical MS position $\boldsymbol{\xi}$, which can be arbitrarily chosen in the particular scenario. All positions are specified in a 2D Cartesian space as follows:

$$\boldsymbol{\xi}_{\mathrm{O}} = \begin{bmatrix} x_{\mathrm{O}} \\ y_{\mathrm{O}} \end{bmatrix}, \quad \boldsymbol{\xi} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \boldsymbol{\xi}_{\mathrm{M}} = \begin{bmatrix} x_{\mathrm{M}} \\ y_{\mathrm{M}} \end{bmatrix}. \tag{1}$$

Let $\mathbf{h}(\boldsymbol{\xi}_{\mathrm{O}}, \boldsymbol{\xi})$ be the function, which represents the result of the RT analysis. For particular transmitter position $\boldsymbol{\xi}$ and receiver poisition $\boldsymbol{\xi}_{\mathrm{O}}$ function $\mathbf{h}$ gives, i.e. RT analysis predicts, a set of multipath parameters consisting of $M_{\boldsymbol{\xi}}$ multipath components:

$$\mathbf{h}(\boldsymbol{\xi}_{\mathrm{O}}, \boldsymbol{\xi}) = \{\mathbf{h}^m(\boldsymbol{\xi}_{\mathrm{O}}, \boldsymbol{\xi})\}_{m=1}^{M_{\boldsymbol{\xi}}}. \tag{2}$$

$\mathbf{h}^m(\boldsymbol{\xi}_{\mathrm{O}}, \boldsymbol{\xi})$ contains parameters describing the m-th multipath. Subscript $\boldsymbol{\xi}$ in $M_{\boldsymbol{\xi}}$ indicates that the number of predicted multipaths can be different for different hypothetical MS positions. For the sake of notation simplicity we use M instead of $M_{\boldsymbol{\xi}}$ whenever there is no danger of ambiguity. For the same reason an explicit specification of OS location can be ignored, i.e. in the sequel we use $\mathbf{h}_{\boldsymbol{\xi}}^m$ instead of $\mathbf{h}^m(\boldsymbol{\xi}_{\mathrm{O}}, \boldsymbol{\xi})$. $\mathbf{h}_{\boldsymbol{\xi}}^m$ has the following structure:

$$\mathbf{h}_{\boldsymbol{\xi}}^m = [l_m \quad \varphi_m]^{\mathrm{T}}, \tag{3}$$

i.e. each multipath component is specified by its relative path length and azimuth DoA. Azimuth DoA is in general bounded by:

$$\varphi_m \in [-\pi, \pi]. \tag{4}$$

l_m is a meter valued parameter, which relates to (the actually observable) *relative* or *excess delay* by:

$$l_m = \tau_m \cdot c_{\mathrm{Light}}. \tag{5}$$

c_{Light} denotes the speed of light. We use l_m instead of τ_m since it is more illustrative and convenient to use a meter valued parameter, while characterizing position estimate. The relative path length l_m is obtained from the complete path length l'_m, by:

$$l_m = l'_m - \min(l'_1, l'_2, \ldots, l'_M). \tag{6}$$

In this way l_m's are calculated from l'_m's also within RT analysis, since latter provides full multipath lengths. With definitions made above l_m is bounded by:

$$l_m \in [0, l_{\max}] , \tag{7}$$

where $l_{\max} = \max(l_1, l_2, \ldots, l_M)$ relates to *total excess delay* $\tau_{\max}$. Let us now define the measured multipath set $\mathbf{z}$ consisting of K multipath components:

$$\mathbf{z} = \left\{\mathbf{z}^k\right\}_{k=1}^{K} . \tag{8}$$

Each measured multipath component $\mathbf{z}^k$ is characterized by its DoA and RToA parameter in the same manner as in the case of predicted multipath from (3). The measurement process incorporates different types of errors, which are introduced in detail in [Alg10] and will be briefly recapitulated here. On the one hand, there are false alarms and missing detections of true multipaths. That is, even if we would know the true MS position $\boldsymbol{\xi}_{\mathrm{M}}$ the number of measured multipaths K and predicted multipaths $M_{\boldsymbol{\xi}_{\mathrm{M}}}$ will be in general different. On the other hand, there is additive Gaussian distortion, which can lead to misassociations especially when the multipaths are closely spaced. All those errors can significantly deteriorate the localization accuracy or may result in completely wrong coordinates. Therefore, a careful choice of an underlying likelihood function is essential.

3 M-type likelihood function

As already mentioned in the introduction we wish to develop a likelihood criterion, which, firstly, requires minor statistical information about the clutter and, secondly, is able to cope with the multipath order problem. Let us demonstrate the problems arising, when the classical definition of the likelihood function is straightforwardly applied to the single observation:

$$\mathcal{L}(\boldsymbol{\xi}) = \prod_{k=1}^{K} \mathcal{N}\left(\mathbf{z}^k; \mathbf{h}_{\boldsymbol{\xi}}^k, \mathbf{C}^k\right) . \tag{9}$$

Hereby $\mathbf{C}^k$ denotes the measurement covariance matrix of the k-th multipath. With noise variances $\sigma_{l_k}^2, \sigma_{\varphi_k}^2$ corresponding to relative path length and DoA respectively, $\mathbf{C}^k$ is defined as follows:

$$\mathbf{C}^k = \mathrm{diag}\left[\sigma_{l_k}^2, \sigma_{\varphi_k}^2\right] . \tag{10}$$

The values of the noise variances are known and depend on the array configuration, system bandwidth, SNR. They are typically different for every measured multipath. However, for the sake of simplicity, we assume equal variances for all multipaths.

According to the *maximum likelihood estimation* principle the location maximizing $\mathcal{L}(\boldsymbol{\xi})$ is the most likely to be the correct MS position estimation, i.e. the corresponding *maximum* *likelihood estimator* (MLE) is expressed as follows:

$$\begin{aligned}\widehat{\boldsymbol{\xi}}_{\mathrm{M}}^{MLE} &= \underset{\boldsymbol{\xi}}{\operatorname{argmax}}\left(\prod_{k=1}^{K}\mathcal{N}\left(\mathbf{z}^k;\mathbf{h}_{\boldsymbol{\xi}}^k,\mathbf{C}^k\right)\right)\\ &\propto \underset{\boldsymbol{\xi}}{\operatorname{argmin}}\left(\sum_{k=1}^{K}\frac{1}{2}\left(\mathbf{h}_{\boldsymbol{\xi}}^k-\mathbf{z}^k\right)^{\mathrm{T}}\left(\mathbf{C}^k\right)^{-1}\left(\mathbf{h}_{\boldsymbol{\xi}}^k-\mathbf{z}^k\right)\right)\\ &= \underset{\boldsymbol{\xi}}{\operatorname{argmin}}\left(\sum_{k=1}^{K}\sum_{i=1}^{R}\frac{1}{2}\frac{\left(\mathbf{h}_{\boldsymbol{\xi}}^k(i)-\mathbf{z}^k(i)\right)^2}{\mathbf{C}^k(i,i)}\right) = \underset{\boldsymbol{\xi}}{\operatorname{argmin}}\left(\sum_{k=1}^{K}\sum_{i=1}^{R}\frac{\left(u_{\boldsymbol{\xi}}^{k,i}\right)^2}{2}\right), \quad (11)\end{aligned}$$

where R denotes the number of parameters describing a single multipath. In our case $R = 2$, however, it changes if further parameters e.g. elevation DoA or Doppler shift are considered. $\mathbf{C}^k(i,i)$ addresses the i-th diagonal element, i.e. the variance of the i-th multipath parameter characterizing the k-th multipath component, while $\mathbf{h}_{\boldsymbol{\xi}}^k(i)$ addresses the i-th entry of the vector $\mathbf{h}_{\boldsymbol{\xi}}^k$. $u_{\boldsymbol{\xi}}^{k,i} = \frac{\left(\mathbf{h}_{\boldsymbol{\xi}}^k(i)-\mathbf{z}^k(i)\right)}{\sqrt{\mathbf{C}^k(i,i)}}$ is the i-th normalized residual of the k-th multipath component for the hypothesized MS position $\boldsymbol{\xi}$.

The aim is to find position $\widehat{\boldsymbol{\xi}}_{\mathrm{M}}$ so that the corresponding multipath parameters $\mathbf{h}_{\widehat{\boldsymbol{\xi}}_{\mathrm{M}}}$ generated via RT yield the best match with the measured parameters $\mathbf{z}$. However, the likelihood function from (9), i.e. the estimator from (11), shows some deficiency in practical use. Firstly, the predicted multipath parameters must be associated with the measured ones, since the relation between them is unknown. Secondly, (9) is inapplicable if the number of predicted multipaths $M_{\boldsymbol{\xi}}$ at the particular position $\boldsymbol{\xi}$ is not equal to the number of measured multipaths K. Whereas in the case with $K < M_{\boldsymbol{\xi}}$ simply the subset of predicted multipaths with the highest weight can be chosen, the opposite case with $K > M_{\boldsymbol{\xi}}$ is more crucial. Typically, the joint weight $\mathcal{L}(\boldsymbol{\xi})$ diminishes with the number of factors considered, since the individual weights are mostly smaller than 1. Such behavior inevitably causes a higher joint weight for positions with smaller $M_{\boldsymbol{\xi}}$ penalizing those with higher $M_{\boldsymbol{\xi}}$ values. However, this fact counteracts the idea that the position with the highest number of associations is most likely to be the correct one. Thirdly, the product structure of $\mathcal{L}(\boldsymbol{\xi})$ makes it very sensitive to small values of the individual weights. Hence a single outlier with the vanishing individual weight decreases drastically the joint weight even if the remaining individual weights are significant.

The first point addressed as *data association* problem can be seen as an independent task and is not discussed in this contribution. The interested reader is referred to [Alg10]. Instead we concentrate in the following on the remaining two points and define the modified likelihood criterion inspired by the concepts of robust statistics.

Classical statistical methods rely heavily on model assumptions which are often not met in practice. E.g. if there are outliers in the data or if the assumption of the noise signal distribution (typically Gaussian) is violated, classical methods often have very poor performance. Robust statistics seeks to provide concepts which describe the behavior of statistical procedures not only under strict parametric models, but also in neighborhoods of such models, see [Hub81]. *M-estimator* ("M" stands for "maximum likelihood-type") is a

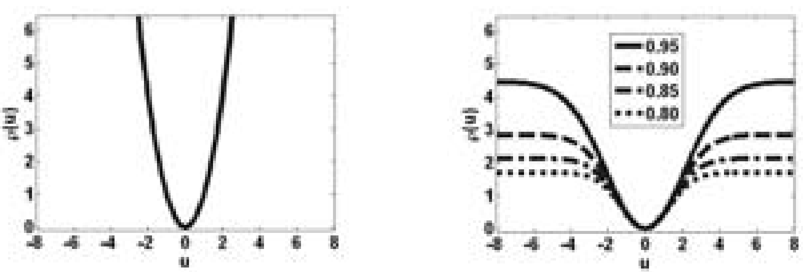
Table 1: Two possible ρ-functions for M-estimators.

Name	$\rho(u)$
Least squares	$u^2/2$
Welsh	$c_\alpha^2/2\left(1-\exp\left(-\left(u/c_\alpha\right)^2\right)\right)$

popular robust technique, which generalizes maximum likelihood estimation from (11) to:

$$\widehat{\boldsymbol{\xi}}_{\mathrm{M}}^{ME} = \underset{\boldsymbol{\xi}}{\operatorname{argmin}} \left(\sum_{k=1}^{K} \sum_{i=1}^{R} \rho\left(u_{\boldsymbol{\xi}}^{k,i}\right) \right). \tag{12}$$

where ρ is a symmetric, positive-definite function, i.e. $\rho(u) > 0$ for all u accept 0, with a unique minimum at zero, i.e. $\rho(0) = 0$. The idea is to choose ρ in such a way to provide the estimator desirable properties in terms of bias and efficiency. There is a wide choice of underlying functions, see [Rey83]. Figure 1 and table 1 present two possible alternatives for one dimensional case. The first candidate in table 1 is the least squares estimator, which corresponds to the maximum likelihood case from (11). Note, that the ρ-function of a least squares estimator is not bounded and residuals cause its quadratic increase, which explains the strong effect of outliers, see figure 1(a).



(a) Least squares.

(b) Welsh; with varying asymptotic efficiency.

Figure 1: Shapes of ρ-functions.

The second candidate is the Welsh function, which is the Gaussian-like curve turned upside down, thus ensuring absolute suppression of outliers lying far from zero, see figure 1(b). This attractive property represents a decisive factor by the choice of an appropriate ρ-function in the context of BML problem. It allows, furthermore, to compare MS locations with different number of detected multipaths.

Using the Welsh function and applying standard statistical calculation the *M-type* likelihood function can be expressed as follows:

$$\mathcal{L}(\boldsymbol{\xi})_{W}^{ME} = \prod_{k=1}^{K} \prod_{i=1}^{R} \frac{\exp\left(-\rho\left(u_{\boldsymbol{\xi}}^{k,i}\right)\right)}{\sqrt{2\pi \mathbf{C}^{k}(i,i)}} \propto \prod_{k=1}^{\min(K,M_{\boldsymbol{\xi}})} \exp\left(\frac{c_\alpha^2}{2} \sum_{i=1}^{R} \exp\left(-\left(\frac{u_{\boldsymbol{\xi}}^{k,i}}{c_\alpha}\right)^2\right)\right). \tag{13}$$

The tuning constant c_α used in the definition of Welsh function allows to adapt the underlying *M-estimator*, i.e. make it either more robust or more efficient. Further details and the derivation of (13) are presented in the full paper.

4 Conclusions

We presented the robust likelihood function for the *blind mobile localization* task. The likelihood function provides a non-linear mapping between the observation and the state space exploiting the knowledge of specific sensor properties combined with RT prediction, thus giving an answer to the fundamental question, how likely a particular hypothetical MS location is to produce an observed set of multipath components. In essence, the underlying likelihood function evaluates the proximity of the measured and predicted multipath components. A special feature of the developed likelihood criterion is that it is able to tackle the *multipath order problem* inevitably arising when positions with different number of multipath components are compared. Furthermore, it provides a robust and reliable position estimate having at hand minimum a priori information about the clutter parameters.

References

[ADKT08] V. Algeier, B. Demissie, W. Koch, and R. Thomä. State Space Initiation for Blind Mobile Terminal Position Tracking. *EURASIP Journal on Advances in Signal Processing*, 2008:14, 2008. Article ID 394219, 14 pages.

[Alg10] Vadim Algeier. *Blind localization of mobile terminals in urban scenarios*. PhD thesis, Ilmenau University of Technology, 2010.

[GG05] F. Gustafsson and F. Gunnarsson. Mobile positioning using wireless networks: possibilities and fundamental limitations based on available wireless network measurements. *IEEE Signal Processing Magazine*, 22(4):41–53, July 2005.

[Hub81] Peter J. Huber. *Robust Statistics*. New York: John Wiley & Sons, 1981.

[KH06] K. W. Kolodziej and J. Hjelm. *Local Positioning Systems: LBS Applications and Services*. Taylor & Francis Group, New York, NY, USA, 2006.

[KST06] Shohei Kikuchi, Akira Sano, and Hiroyuki Tsuji. Blind Mobile Positioning in Urban Environment Based on Ray-Tracing Analysis. *EURASIP Journal on Applied Signal Processing*, 2006:12, 2006. Article ID 38989, 12 pages.

[Mau05] Jürgen Maurer. *Strahlenoptisches Kanalmodell für die Fahrzeug-Fahrzeug-Funkkommunikation*. PhD thesis, Universität Karlsruhe, 2005.

[Rey83] W. J. J. Rey. *Introduction to Robust and Quasi-Robust Statistical Methods*. Berlin Heidelberg: Springer-Verlag, 1983.