Sensor Resolution Models and Multidimensional Data Association

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Abstract: The multidimensional data association methods were developed to establish the relation between plots and tracks especially in dense target situations. However, even these advanced multidimensional data association methods lack in situations of unresolved measurement. Specifically, in real dense target situations being of most interest the phenomena of unresolvedplots happens quite often due to the limited sensor resolution.

The data association describes the correlation between tracks representing targets and plots. There are several hypotheses about the origin of a new sensor plot: It may be unreal and has to be ignored (false alarm). Otherwise, it is assumed to be real and then one has to discriminate between plots related to tracks supported by previous plots (update) and those, which are not (new track). Finally, an existing track may be not reported due to a miss (no detection). To handel also unresolved plot hypotheses a sensor resolution model is necessary which has to be integrated into the multidimensional data association framework.

1 Resolution Models

The resolution of a radar system means the distance in any dimension between two targets so that they are reported as independent plots. One has to find a heuristic model, which allows an overall approximation of such effects.

1.1 Linear Model

The most simple class of models are the linear ones [BP99]. Let $w_i, i \in \{1,\dots,L\}$ be the distance between the targets t_1 and t_2, e.g. in range azimuth, elevation or Doppler and d_i^R the sensor resolution. One postulates for the merging probability wrt. the component i:

$$P_i^M(t_1,t_2)=\begin{cases}0; & \text{if } w_i \ge d_i^R \\ \dfrac{d_i^R - w_i}{d_i^R}; & \text{if } w_i < d_i^R\end{cases} \text{ and } P^M(t_1,t_2)=\prod_{i=1}^{L} P_i^M(t_1,t_2) \tag{1}$$

1.2 Gaussian Model

W. Koch and G. van Keuk describe the merging by a Gaussian approach [KK97].

$$P^M(t_1,t_2)=\exp\left(-\tfrac{1}{2}w^{\mathrm{T}}Dw\right) \tag{2}$$

This model takes care of a ellipsoidal resolution cells expressed by the matrix D .

1.3 Resolution-Cell Model

Another model developed in [CB84], [BL95] takes care of the standard deviation of the estimated distances.

2 Filtering

The assumed dynamics of target t is the constant velocity model. I.e., the dynamical system is defined by

$$x_k^t = F_{k-1}^t(x_{k-1}^t) + q_{k-1}^t \text{ and } z_k^j = H_k^t(x_k^t) + r_k^j \tag{3}$$

Here x_k^t stands for the target state vector and q_k^t describes a Gaussian process noise of covariance Q_k^t . The vector of a sensor plot j is written as z_k^j . This measurement is related to the actual target state by with a measurement noise r_k^j defined as Gaussian distributed with covariance R_k^j . One uses the Kalman filtering as in [CB84] to update the target states with resolved and unresolved plots. The new state for the track t_1 is defined by

$$x_k^{t_1} = \begin{cases} x_{k|k}^{t_1 j} \\ x_{k|k}^{t_1 t_2 j} \end{cases} \text{ and } P_k^{t_1} = \begin{cases} P_{k|k}^{t_1 j} & \text{single update with plot } j \\ P_{k|k}^{t_1 t_2 j} & \text{cluster update with } t_2 \text{ and plot } j \end{cases} \tag{4}$$

3 Data association with unresolved plots

The basic assumption for the association process is, that a plot of one scan is either resolved and originates from one target or is a false alarm. Or it is unresolved and belongs to a cluster of exactly two targets. On the other side a track is updated by one (unresolved or resolved) plot of each scan or is not detected within a scan. The symbol 0 indicates the false plot hypothesis. To include the hypothesis of clusters consisting of two targets, one uses the following encoding: Here a combination $(t_1, t_1) \in T^2$ stands for a single track t_1, which is reported by a single resolved plot. A pair (t_1, t_2) with $0 \neq t_1 \neq t_2 \neq 0$ determines track t_1, which is assumed in a 2-cluster with track t_2. The symbol $(0,0)$ determines the false alarm hypothesis for a given plot. The data association problem is now translated into an integer linear programming problem. Let $I_1 = \{0,\ldots,n_1\}$ determine the (unresolved or resolved) plots of the full further scan, where 0 symbolized a missed detection. One obtains the optimization problem:

$$\min \sum_{t_1,t_2=0}^{m} \sum_{i_1=0}^{n_1} c_{t_1 t_2 i_1} \chi_{t_1 t_2 i_1} \tag{5}$$

Here $\chi_{t_1 t_2 i_1} \in \{0,1\}$ is an indicator function, such that $\chi_{t_1 t_2 i_1} = 1$ selects the association between the cluster (t_1, t_2) and the measurement i_1. The $c_{t_1 t_2 i_1}$ are the negative logarithms of the association probabilities, $c_{t_1 t_2 i_1} = -\ln(L_{t_1 t_2 i_1})$. The indicator function is subject to the following constraints: The first constraint expresses the symmetry of the cluster definition. If target t_1 forms a cluster with target t_2 and has therefore a common unresolved plot i_1, then also target t_2 forms a cluster with target t_1 with common plot i_1, i.e.

$$\chi_{t_1 t_2 i_1} = \chi_{t_2 t_1 i_1}; t_1 < t_2; t_1, t_2 = 0,\ldots,m; i = 0,\ldots,n_1 \tag{6}$$

Each measurement is either unresolved and belongs to exactly two tracks, or is resolved and is a false alarm or belongs to a single track. Further, each track can be updated by at most one plot:

$$\sum_{t_1=0}^{m} \sum_{t_2=t_1}^{m} \chi_{t_1 t_2 i_1} = 1; i_1 = 1,\ldots,n_1 \text{ and } \sum_{i_1=0}^{n_1} \sum_{t_2=0}^{m} \chi_{t_1 t_2 i_1} = 1; t_1 = 1,\ldots,m \tag{7}$$

A regular update of a single track with a resolved plot resp. missed detection adds the following weight to the tracks score:

$$L_{t_1 t_1 i_1} = [1 - P_D]^{\delta_{i_1,0}} [N(z_k^{i_1}; z_{k|k-1}^{t_1 i_1}, S_k^{t_1 i_1}) \frac{P_D}{\rho_F} \prod_{\substack{j=1 \\ j \neq t_1}}^{m} (1 - P_M(t_1, j))]^{1-\delta_{i_1,0}}; \qquad (8)$$

Finally, the weight for an unresolved plot is determined by:

$$L_{t_1 t_2 i_1} = N(z_k^{i_1}; z_{k|k-1}^{t_1 t_2 i_1}, S_k^{t_1 t_2 i_1}) \frac{P_D}{\rho_F} P_M(t_1, t_2) \prod_{\substack{j=1 \\ j \neq t_1, t_2}}^{m} (1 - P_M(t_1, j)); \qquad (9)$$

4 The optimization problem

Through the above treatment, the data association problem is transformed into an integer Linear Programming. There are several mathematical approaches which address this problem. One of them is to preliminarily ignore the integer constraint $\chi_{t_1 t_2 i_1} \in \{0,1\}$ of the indicator function and use $0 \leq \chi_{t_1 t_2 i_1} \leq 1$ instead. Therefore, one is able to apply Linear Programming techniques. Especially the homogeneous self dual interior point method [YTM94] can be used to handle this problem, as proposed by [LLW99]. The result $p_{t_1 t_2 i_1}$ is interpreted as pseudo probabilities similar to a JPDA approach i.e.

$$p_{t_1 t_2 i_1} = \mathrm{Prob}(\chi_{t_1 t_2 i_1} = 1) \qquad (10)$$

This means $p_{t_1 t_2 i_1}$ is the probability that track t_1 forms a cluster with track t_2 with unresolved plot i_1 ($t_1 \neq t_2$) resp. the probability that track t_1 is separated and is updated with the resolved plot i_1 ($t_1 = t_2$). This interpretation makes sense due to the constraints.

5 Simulation Results

To illustrate the performance of the described methods some evaluations are given below. For the solution of the optimisation steps the homogeneous self dual interior point method was used.

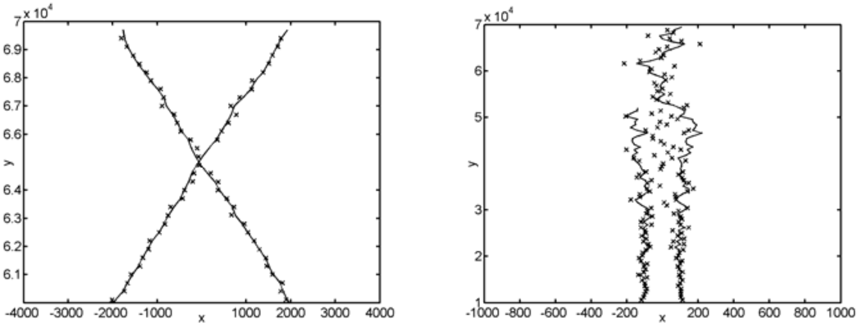

Figure 1. Two target scenarios with unresolved plots.

The first picture shows a crossing scenario of two targets with unresolved measurements in the crossing area. processed by the modified two dimensional approach. One realizes, that both tracks are continuously tracked, even in this region. The second one demonstrates two targets coming from outbound. This scenario is distinguished by the early track initialisation specifically in the region, where the sensor is not able to resolve the target situation continuously.

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